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Additional laws and theorem of Boolean Algebra

Algebra :- Like ordinary algebra, commutative, associative & distributive laws are verified for Boolean Algebra.

~ Commutative laws :- $A + B = B + A$
 $A \cdot B = B \cdot A$

~ Associative laws :- $A + (B + C) = (A + B) + C$
 $A (BC) = (AB) C$

~ Distributive laws :- $A (B + C) = AB + AC$

Other basic relations valid for Boolean Algebra

OR
 $A + 0 = A$ $A + A = A$
 $A + 1 = 1$ $A + \bar{A} = 1$

AND
 $A \cdot 0 = 0$ $A \cdot A = A$
 $A \cdot 1 = A$ $A \cdot \bar{A} = 0$

These two groups of law can be thought of in terms of OR and AND gates. Each of these identities can be proved, by substituting the two possible values of A i.e. 0 and 1; on each side of the identity. In each case, the left-hand side will equal to the right-hand side. Again, the double-complement operation gives $\bar{\bar{A}} = A$.

→ Some other useful theorem:-

$$(i) \quad A + AB = A$$

$$(ii) \quad A + \bar{A}B = A + B$$

$$(iii) \quad A(A+B) = A$$

$$(iv) \quad A(\bar{A}+B) = AB$$

$$(v) \quad (A+B)(A+C) = A+BC$$

$$(vi) \quad (A+B)(\bar{A}+C) = AC + \bar{A}B$$

Putting the two values of 0 & 1; above identities can be proved:-

⇒ Circuit Representation of De-Morgan's first theorem:-

(a) NOR Gate (b) NAND Gate ($\overline{A \cdot B}$)

De-Morgan's first theorem $\overline{A+B} = \overline{A} \cdot \overline{B}$; suggests that a logic system in which a NOT gate follows an OR gate and is called NOT-OR or NOR gate (shown Fig 4(a), (b)).

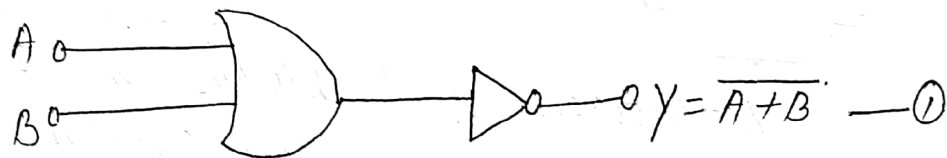


Fig 4(a) NOT-OR gate

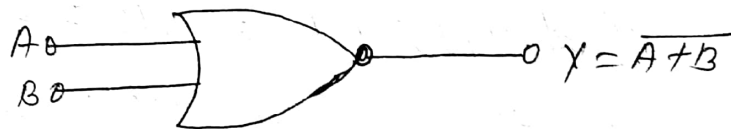
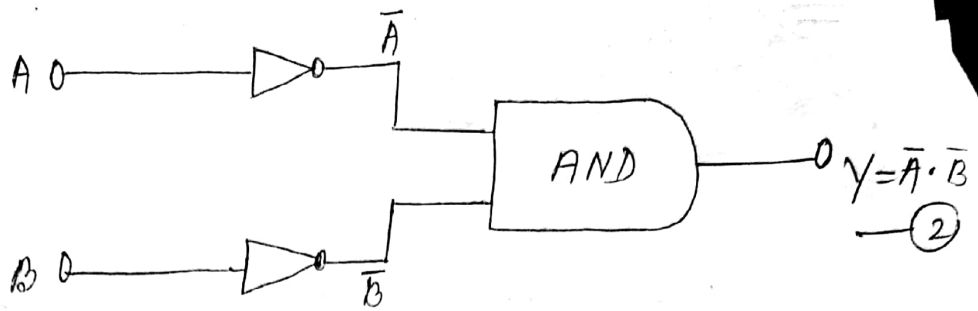


Fig 4(b) NOR-gate

The truth table of two input NOR gate is given below:-

Input		output
A	B	$Y = \overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

(b) This NOR-gate is also equivalent to a logic system in which the inputs to an AND gate are the outputs of two NOT-gates shown in fig 4(c).



Here Fig- 4(c).
 Fig. 4(a) & Fig 4(c) are the representation of D.M. 1st theorem.

⇒ Circuit Representation of D.M. 2nd theorem

D-Morgan's 2nd theorem is $\overline{A \cdot B} = \overline{A} + \overline{B}$
 It indicates that a logic system in which NOT CRT. follows an AND gate is called NAND gate or NOT-AND gate. shown in fig. 5(a), (b) which is equivalent to a logic.

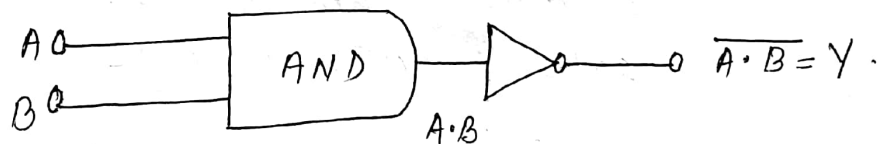


Fig- 5(a) :

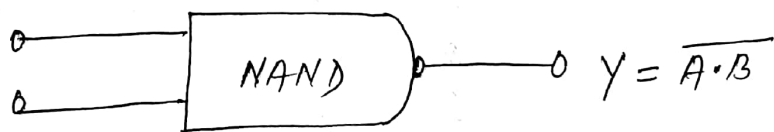
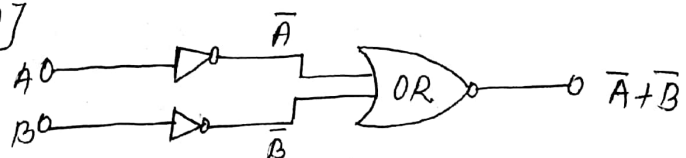


Fig- 5(b) :

System in which the inputs of an OR gates are the outputs from two NOT CRTs [shown in fig- 5(c)]



The truth table of NAND gate is shown below:-

inputs		output	input		output
A	B	$Y = A \cdot B$			
0	0	1	1	0	0