

Principle of Virtual work

This principle states that "when under the action of any forces, a system of particles is in stable equilibrium, and this system receives or is imagined to ~~be~~ receive, any small disturbance, the total amount of work done by all the forces acting on the various particles is zero." In other words, the potential energy of the system is a minimum.

Let us apply this principle to the equilibrium of a homogeneous liquid at rest in a vessel under the action of gravity. An elementary volume, $dx dy dz$ of the liquid are taken at a height z as shown in fig (1).

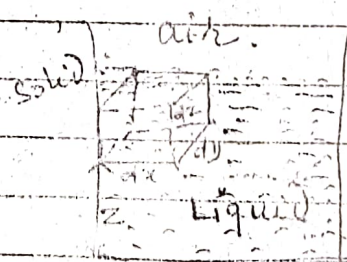


Fig (1)

The potential energy of the whole system is composed of four parts.

(1) Gravitational energy equal to

$$\rho g \iiint z \, dx dy dz$$

$$(\cancel{Mgh})$$

$$(\because P.E = V \rho g h) \\ = m g h$$

(b) Energy of the liquid-air surface
i.e. AS_1

(c) Energy of the ~~air~~ liquid-solid surface
i.e. BS_2 and

(d) Energy of the air-solid surface.
i.e. CS_3

Hence total potential energy

$$g \iiint z \, dx \, dy \, dz + AS_1 + BS_2 + CS_3$$

must be minimum.

Where A, B, C represent the area of the surfaces (b), (c) & (d) and S_1, S_2 and S_3 are the surface tensions.

Now consider only surface A changes from A to A_1 in the direction normal to ~~the~~ it where as B and C remains constant. The change in area changes the potential energy as

$$g \iint z \cdot \delta l \, dA \quad \text{and} \quad \rightarrow (1)$$

where $dA = dx \, dy$ and $dz = \delta l$.

The change in area A is given by

$$\delta(dA) = dA_1 - dA \quad \text{as clear from fig 2.}$$

$$= dh' \, dh_2 - dh \, dh_2 \quad \rightarrow (2)$$

The value of dh' & dh_2' are obtained with the help of fig (3).

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If r_1 and r_2 represent the principal radii of curvature of the arcs dl_1, dl_2 ; then for arc dl_1 & dl_1' , we have

$$\theta = \frac{dl_1}{r_1} = \frac{dl_1'}{r_1 - \delta l}$$

$$\therefore dl_1' = \left(\frac{r_1 - \delta l}{r_1} \right) dl_1$$

$$= \left(1 - \frac{\delta l}{r_1} \right) dl_1$$

$$\text{Similarly } dl_2' = \left(1 - \frac{\delta l}{r_2} \right) dl_2$$

putting these values in eqⁿ (2), we have

$$\delta(dA) = - \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \delta l dA$$

from the principle of Virtual work

$$g \rho \iint z \delta l dA + S_1 \iint \delta(dA) = 0$$

$$\text{or, } \iint \left[g \rho z - S_1 \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right] \delta l dA = 0 \rightarrow (4)$$

Taking volume remains constant

$$\therefore \iint \delta l dA = 0 \rightarrow (5)$$

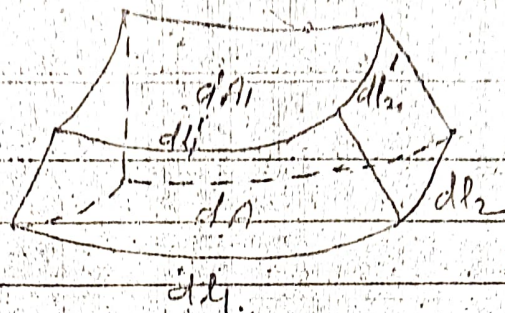


Fig (2)

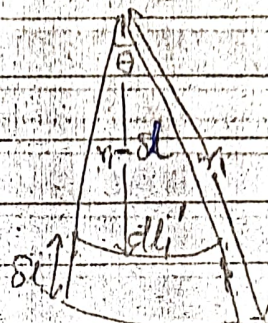


Fig (3)

$$\therefore \iint \left[g \rho z - S_1 \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right] \delta l dA = 0 \quad \text{--- (4)}$$

Multiplying above equation by constant term $-k\rho g$, we have

$$-k\rho g \iint \delta l dA = 0 \quad \text{--- (5)}$$

where k is taken as height of atmosphere from the earth.

Adding eqⁿ (5) to eqⁿ (4) we have

$$\iint \left[g \rho (z-k) - S_1 \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right] \delta l dA = 0$$

On integrating, we have

$$S_1 \left(\frac{1}{r_1} + \frac{1}{r_2} \right) = g \rho (z-k) \quad \text{--- (6)}$$

$$\text{or } S_1 \left(\frac{1}{r_1} + \frac{1}{r_2} \right) = p - p_0 \quad \text{--- (7)}$$

this is the required expression for excess pressure.

$$* S_1 \left(\frac{1}{r_1} + \frac{1}{r_2} \right) = p + \text{constant}$$

where p is the pressure just within the surface of liquid. If the left-hand side is zero - which is the case when the surface is flat - then $p = p_0$, where p_0 is the atmospheric pressure. Hence finally,