* Newton's Divided difference interpolation formula is a interpolation method/technique, it is used when the interval difference is not same for all sequence of values.

Let $(x_0, y_0), (x_1, y_1) \cdots (x_n, y_n)$ be the given $(n+1)$ points. Then the divided differences of order $1, 2, \cdots n$ are defined by the relations:

$$(1) \quad \begin{cases} [x_0, x_1] = \dfrac{y_1 - y_0}{x_1 - x_0} \\ [x_0, x_1, x_2] = \dfrac{[x_1, x_2] - [x_0, x_1]}{x_2 - x_0} \\ \vdots \\ [x_0, x_1, x_2 \cdots x_n] = \dfrac{[x_1, x_2, \cdots x_n] - [x_0, x_1, \cdots x_{n-1}]}{x_n - x_0} \end{cases}$$

Even if the arguments are equal, the divided differences may still have a meaning. We then set

$$x_1 = x_0 + \epsilon$$

$$\therefore [x_0, x_1] = \lim_{\epsilon \to 0} [x_0, x_0 + \epsilon]$$

$$= \lim_{\epsilon \to 0} \frac{y(x_0 + \epsilon) - y(x_0)}{\epsilon}$$

$$= y'(x_0), \text{ if } y(x) \text{ is differentiable.}$$

Similarly,

$$[x_0, x_1, x_2, \cdots x_n] = \frac{y^r(x_0)}{\lfloor r}$$

($r+1$ arguments)

From (1), it is easy to see that

$$[x_0 x_1] = \frac{y_0}{x_0 - x_1} + \frac{y_1}{x_1 - x_0} = [x_1, x_0]$$

$$[x_0, x_1, x_2] = \frac{1}{x_2 - x_0}\left\{\left[\frac{y_2}{x_2 - x_1} - y_1\left(\frac{1}{x_2 - x_1} + \frac{1}{x_1 - x_0}\right)\right] + \frac{y_0}{x_1 - x_0}\right\}$$

$$= \frac{y_0}{(x_0 - x_1)(x_0 - x_2)} + \frac{y_1}{(x_1 - x_0)(x_1 - x_2)} + \frac{y_2}{(x_2 - x_0)(x_2 - x_1)}$$

$$[x_0, x_1, \cdots x_n] = \frac{y_0}{(x_0 - x_1)\cdots(x_0 - x_n)} + \frac{y_1}{(x_1 - x_0)\cdots(x_1 - x_n)} + \cdots + \frac{y_n}{(x_n - x_0)(x_n - x_1)\cdots(x_n - x_{n-1})}$$

Hence the divided differences are symmetrical in their arguments.

Now let the arguments be equally spaced so that $x_1 - x_0 = x_2 - x_1 = x_3 - x_2 = \cdots = x_n - x_{n-1} = h$

Then $[x_0, x_1] = \frac{y_1 - y_0}{x_1 - x_0} = \frac{1}{h}\Delta y_0$

$$[x_0, x_1, x_2] = \frac{[x_1 - x_2] - [x_0, x_1]}{x_2 - x_0}$$

$$= \frac{1}{2h}\left[\frac{\Delta y_1}{h} - \frac{\Delta y_0}{h}\right]$$

$$= \frac{1}{2h^2}[\Delta y_1 - \Delta y_0] = \frac{1}{2h^2}\Delta^2 y_0$$

$$[x_0, x_1 x_2] = \frac{1}{\lfloor 2 h^2}\Delta^2 y_0$$

in general

$$[x_0, x_1, x_2; \cdots x_n] = \frac{1}{\lfloor n \cdot h^n}\Delta^n y_0$$

If the tabulated function is a polynomial of nth degree, then $\Delta^n y_0$ should be a constant and hence the nth divided difference would also be a constant.