* Differential Eputions of first order and higher degree:-

As $\frac{dy}{dx}$ will occur in higher degrees (or powers), it is denoted as $\frac{dy}{dx} = p$. Such eputions are of the form $f(x, y, p) = 0$.

Three cases arise for discussion:-

Case-I <u>Equation solvable for p</u>:- A differential equation of the first order but of the nth degree is the form -

$$p^n + P_1 p^{n-1} + P_2 p^{n-2} + \cdots \cdots P_n = 0 \quad \text{— (1)}$$

Hence - $f(x, y, p) = 0$

where $P_1, P_2, \cdots \cdots P_n$ are functions of x and y.

Splitting up the left hand sides of Eq(1) into n linear factors, we can write differential Eqn.

$$[p - f_1(x,y)]\,[p - f_2(x,y)] \cdots \cdots [p - f_n(x,y)] = 0$$

Equating each of the factors to zero

$$p - f_1(x,y) = 0 \Rightarrow p = f_1(x,y)$$

$$p - f_2(x,y) = 0 \Rightarrow p = f_2(x,y)$$

$$\vdots$$

$$p - f_n(x,y) = 0 \Rightarrow p = f_n(x,y)$$

Thus $p = f_i(x,y)$, where $i = 1, 2, \cdots n$. which is Linear differential eqution of the first order and first degree.

Suppose solution of $p - f_i(x, y) = 0$ is given by $F_i(x, y, c_i) = 0$, where c_i is an arbitrary constant. Instead of taking different c_i's in the general solution of $p - f_i(x, y) = 0$, if we take only one c in all then it makes no difference in general solution. Therefore general Solution of $p - f_i(x, y) = 0$ is $F_i(x, y, c)$ then

$$F_1(x, y, c) = 0,\ F_2(x, y, c) = 0, \ldots\ F_n(x, y, c) = 0$$

These n solutions constitute the general solution of Eqn (1).

$$F_1(x, y, c) \cdot F_2(x, y, c) \cdot \ldots\ F_n(x, y, c) = 0$$

<u>Exp.</u> Solve $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$

Given solution is $p - \frac{1}{p} = \frac{x}{y} - \frac{y}{x}$ where $p = \frac{dy}{dx}$

or $p^2 + p\left(\frac{y}{x} - \frac{x}{y}\right) - 1 = 0$

factorising $\left(p + \frac{y}{x}\right)\left(p - \frac{x}{y}\right) = 0$

Thus we have

$$p + \frac{y}{x} = 0 \quad \text{---} (i)$$

$$p - \frac{x}{y} = 0 \quad \text{---} (ii)$$

From (i) $\frac{dy}{dx} + \frac{y}{x} = 0$ or $x\,dy + y\,dx = 0$

$d(x y) = 0$

Integrating $xy = c \Rightarrow xy - c$

From (ii) $\frac{dy}{dx} - \frac{x}{y} = 0$ $x\,dx - y\,dx \Rightarrow x^2 - y^2 = c$

Thus Required Solution $(xy - c) \cdot (x^2 - y^2 - c) = 0$ Ans