Paper 1, TDC Part-1
Chapter– 4, Circuit Theorems
Lecture - 8

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Circuit Theorem

Today we will see few more problems on Maximum power transfer theorem.

Q 1)

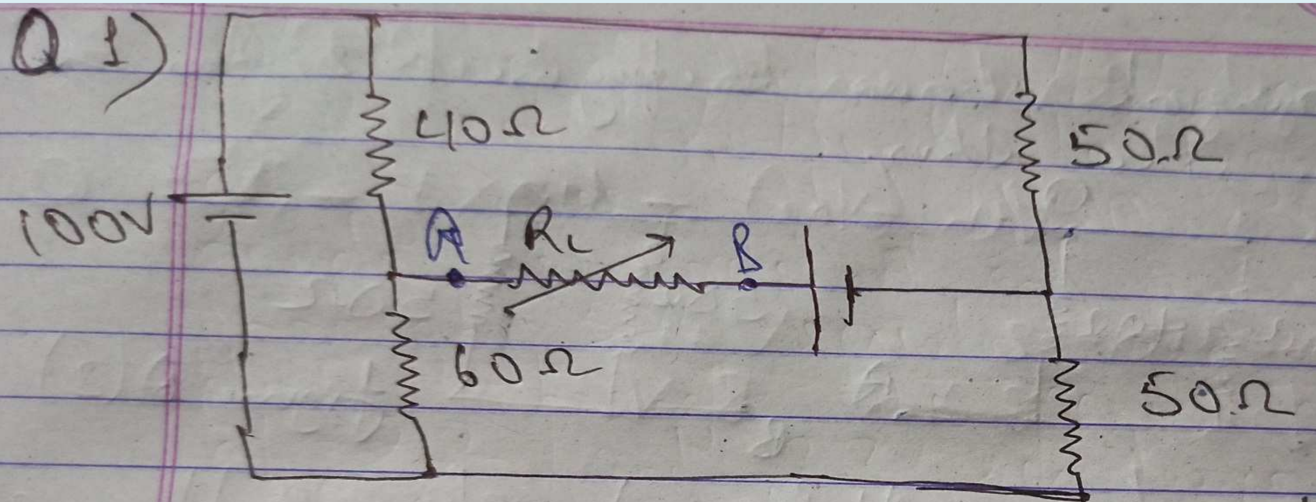

fig A(1)

R_L is variable resistor, ~~whose~~ for what value of R_L maximum power will transfer and also find P_{max}?

Soln. As we know that for maximum power transfer R_L should equal to R_{th}.

Circuit Theorem

As we know that for maximum power transfer R_L should equal to R_{th}.

So to calculate R_{th} we will look in to the circuit from terminal AB by removing R_L as shown below. We will also short circuit the voltage sources and open circuit the current sources.

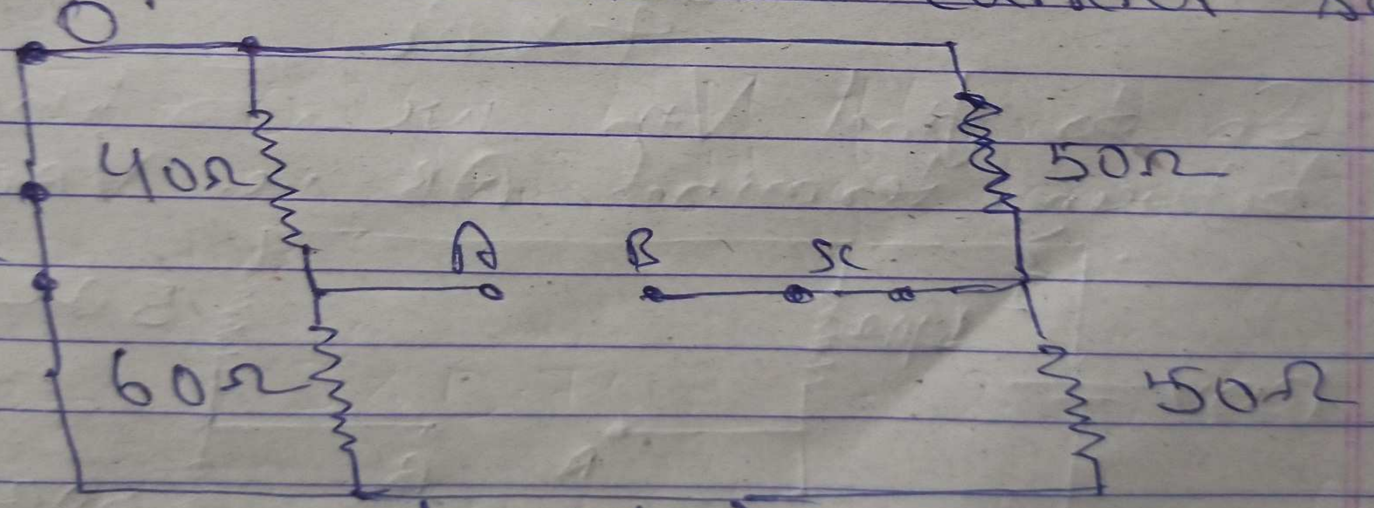

fig A(2)

Circuit Theorem

The network shown in figure A(2) can be seen like this.

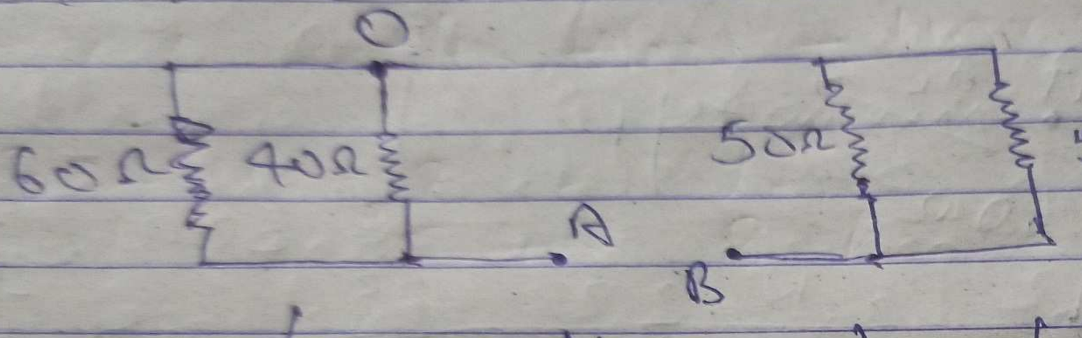

figure A(3)

So $R_{th} = (60\Omega \,\|\, 40\Omega) + (50\Omega \,\|\, 50\Omega)$

$$= \frac{6\cancel{0}\times 4\cancel{0}}{1\cancel{00}} + \frac{5\cancel{0}\times 5\cancel{0}}{1\cancel{00}}$$

$$= 24 + 25\,\Omega$$

$$= 49\,\Omega$$

So for maximum power ~~tha~~ to transfer the value of R_L should be $49\,\Omega$.

Circuit Theorem

= 29 Ω

So for maximum power ~~that~~ to transfer the value of R_L should be 29 Ω.

Now to find P_{max} we ~~should 1st~~ have the

$$P_{max} = \frac{V_{th}^2}{4 \times R_L}$$

So to find V_{th} we have to find voltage between terminal AB i.e. V_{AB}.

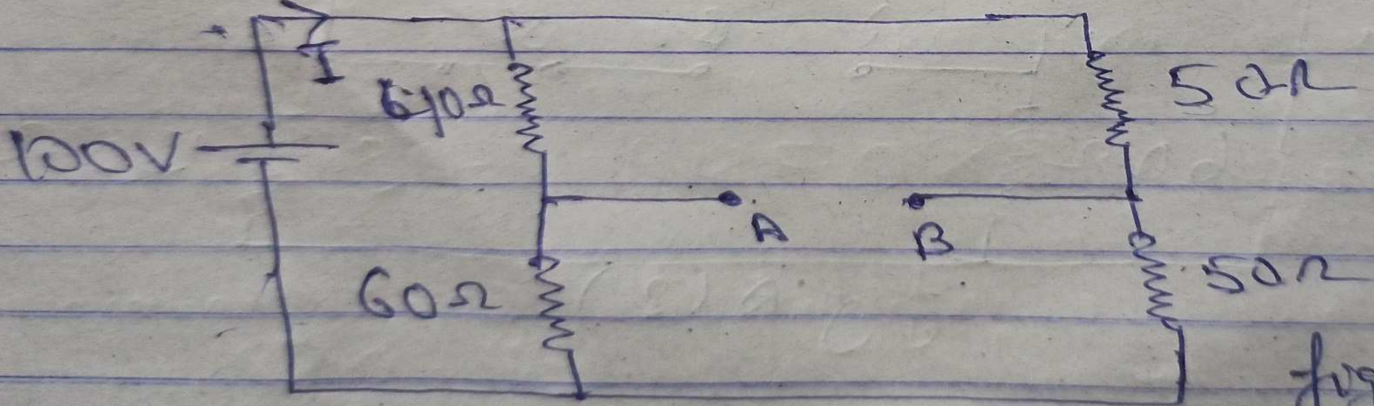

fig (a)

Circuit Theorem

As from the circuit shown in figure A(4) we see that terminal A B is open so no current will flow and hence the resistors 40 & 60 Ω are in series. Illy 50Ω & 50Ω are in series. so the ckt. can be,

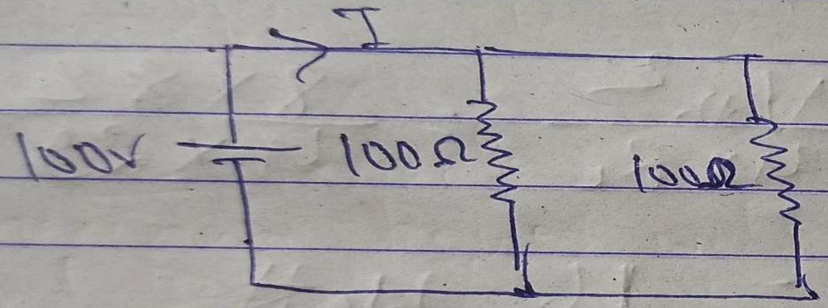

figure A(3)

We see that now 100Ω & 100Ω are ||el.
So,

$$I = \frac{100\ V}{\left(\frac{100\times100}{100+100}\right)\Omega} = \frac{2\cancel{00}\times\cancel{100}}{\cancel{100}\times\cancel{100}}\ A$$

$$I = 2\ A$$

Circuit Theorem

$I = 2A$

This current will distribute equal among the $(40\Omega \parallel 60\Omega)$ & $(50\Omega \parallel 50\Omega)$.

So current ~~following~~ flowing through 40Ω & 50Ω resistor are 1A. as shown in figure below

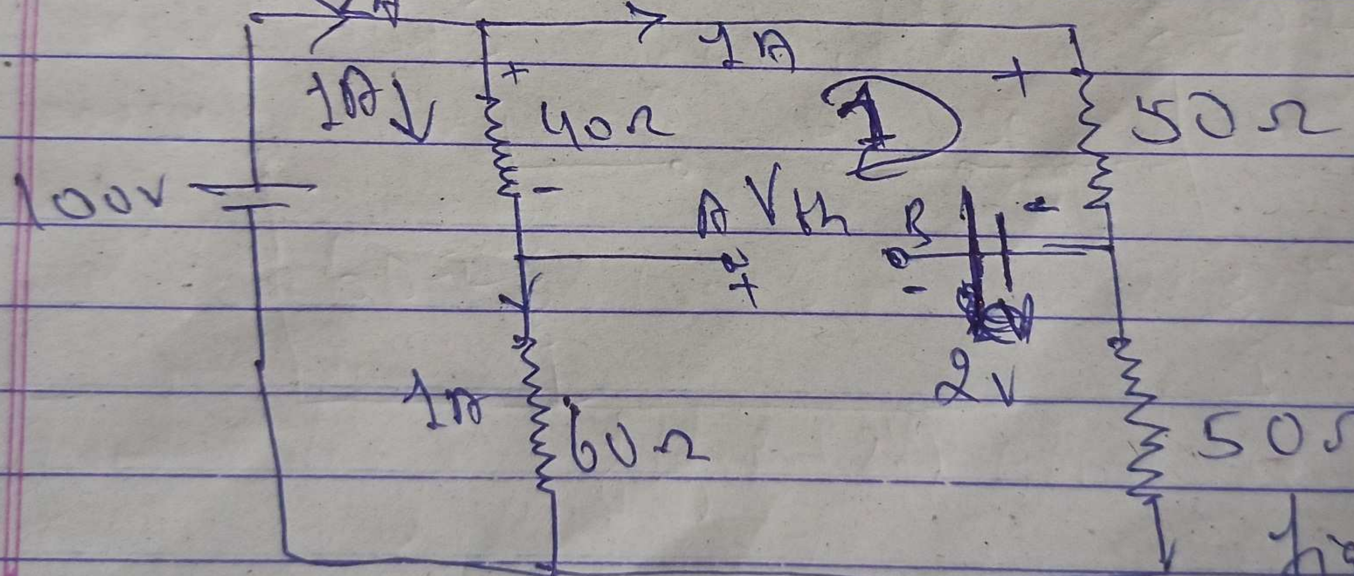

figure A(6)

Circuit Theorem

Applying KVL in loop 1 of figure A(a)
we have,

$$50 \times 1 - V_{th}\,(V_{AB}) - 40 \times 1 - 2 = 0$$

or $50 - 40 - 2 = V_{th}$

or $V_{th} = \cancel{8} \; 8\,V.$

So $P_{max} = \dfrac{V_{th}^2}{4 \times R_L} = \dfrac{8 \times \cancel{8}\,2}{\cancel{4} \times 49} =$

$$= \frac{16}{49} \text{ Watt.}$$

Circuit Theorem

Q2) Find the value of R_L for which the n/w shown below provide maximum power to load resistor. Also calculate P_{max}?

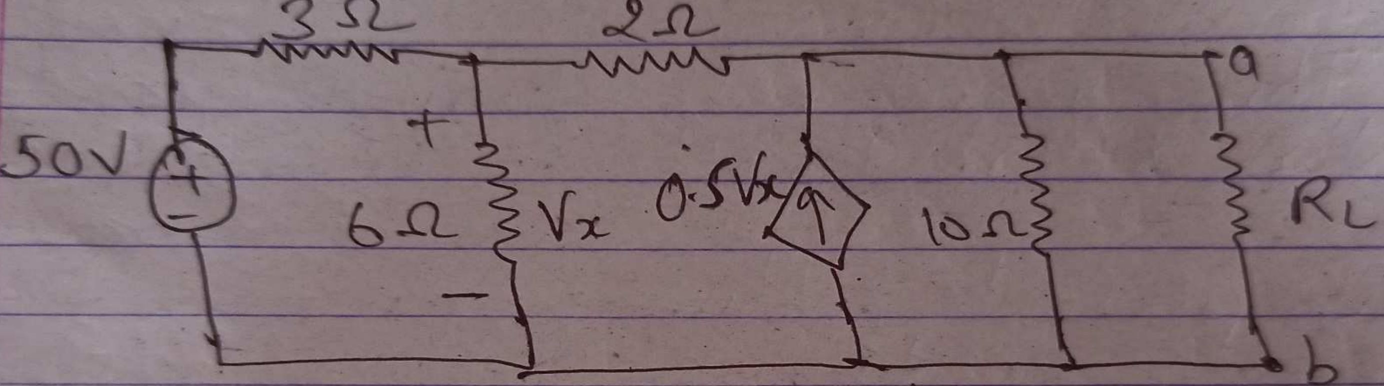

figure B(1)

Soln: As we know that for maximum power to transfer by the n/w, load resistor R_L is,

$$R_L = R_{th}$$

As the n/w consists of voltage dependent current source so we use the following

Circuit Theorem

formula, $R_{th} = \frac{V_{th}}{I_N}$ ~~, where V_{th} is~~

where V_{th} is Thevenin's voltage and
I_N is Norton's current.

but let's see here, to find I_N we have to short terminal a & b by removing R_L.

Now, drawing n/w by short circuiting the terminal AB.

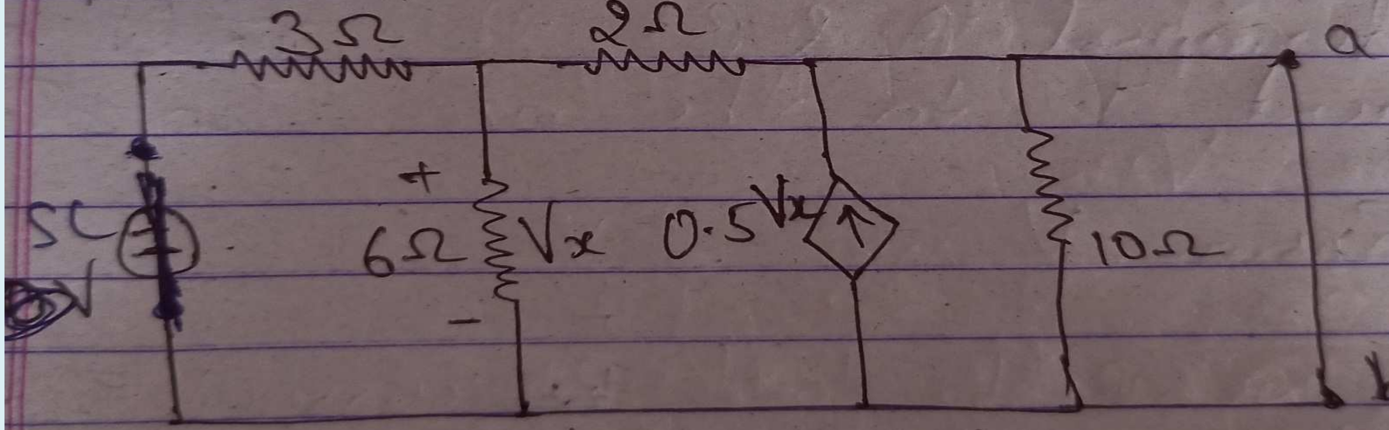

figure B(2)

figure B(2)

Now as we have shorted terminal a&b so the resistor of 10Ω which is $\|$el to terminal a&b will become inactive so no current flow through this and also voltage dependent current source

$0.5V_x$ will become inactive so we can not ~~use~~ short terminal a&b. So to find V_{th} we have ~~use~~ to use another method here that is ~~source~~ we will use test source at terminal a&b and draw the circuit.

Circuit Theorem

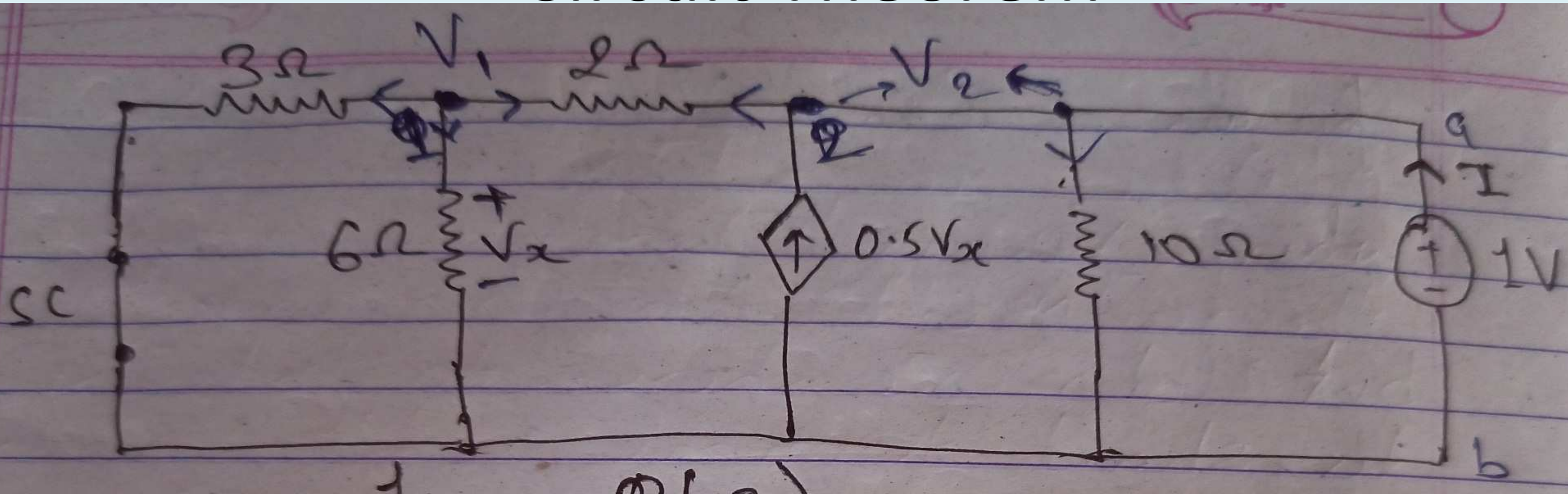

figure B(3)

Here we have connected test source of '1V' at terminal A & B, this will deliver current I to the n/w, so R_{th} will be,

$$R_{th} = \frac{V}{I} = \frac{1\,V}{I}$$

Now

Circuit Theorem

$$R_{th} = \frac{V}{I} = \frac{1\,V}{I}$$

Now we will find ~~Rth~~ current I, for that we will use node analysis method here, for that we have 2 nodes here. where we can apply KCL, let the voltage at terminals are V_1 & V_2 as shown in figure B(3).

Applying KCL at node 1, here all current are out going so, we can write equation as below,

$$\frac{V_1}{3} + \frac{V_1}{6} + \frac{V_1 - V_2}{\cancel{6+}2} = 0 \quad \text{—(i)}$$

and

V_1 is the voltage drop in 6Ω

Circuit Theorem

resistor which is V_x so,

$$V_1 = V_x$$

and voltage at node 2 i.e. V_2 is voltage between terminal a & b ; So,

$$V_2 = 1\,V.$$

putting value of V_1 & V_2 in eqn. (1) we get,

$$\frac{V_x}{3} + \frac{V_x}{6} + \frac{V_x - 1}{2} = 0$$

$$\Rightarrow \frac{2V_x + V_x + 3V_x - 3}{6} = 0$$

$$\Rightarrow 6V_x - 3 = 0$$

$$V_x = \frac{3}{6} = 0.5\,V.$$

Circuit Theorem

$$V_x = \frac{3}{6} = 0.5\,V.$$

Now applying KCL at node 2, here two current is outgoing one is towards 2Ω & other is in 10Ω resistor, while there are 2 incoming currents I & $0.5V_x$ so

KCL at node 2 is,

$$\frac{V_2 - V_1}{2} + \frac{V_2}{10} = 0.5V_x + I$$

$$\frac{1 - V_x}{2} + \frac{1}{10} = 0.5V_x + I$$

$$\frac{1 - 0.5}{2} + 0.1 = 0.5 \times 0.5 + I$$

$$\Rightarrow I = \cancel{0.25} + 0.1 - \cancel{0.25} = 0.1\,A$$

Circuit Theorem

So, $R_{th} = \frac{1}{i} = \frac{10}{0.1} = 10\,\Omega$

So the value of $R_L = R_{th} = 10\,\Omega$ for maximum power to transfer,

Now, $P_{max} = \frac{V_{th}^2}{4 \times R_L}$

To find V_{th} we again draw the ckt. of figure B(1) by removing R_L and find open ~~d~~ terminal voltage at ~~de~~ A & B,

$V_{AB} = V_{th}$,

Circuit Theorem

$V_{AB} = V_{th}$.

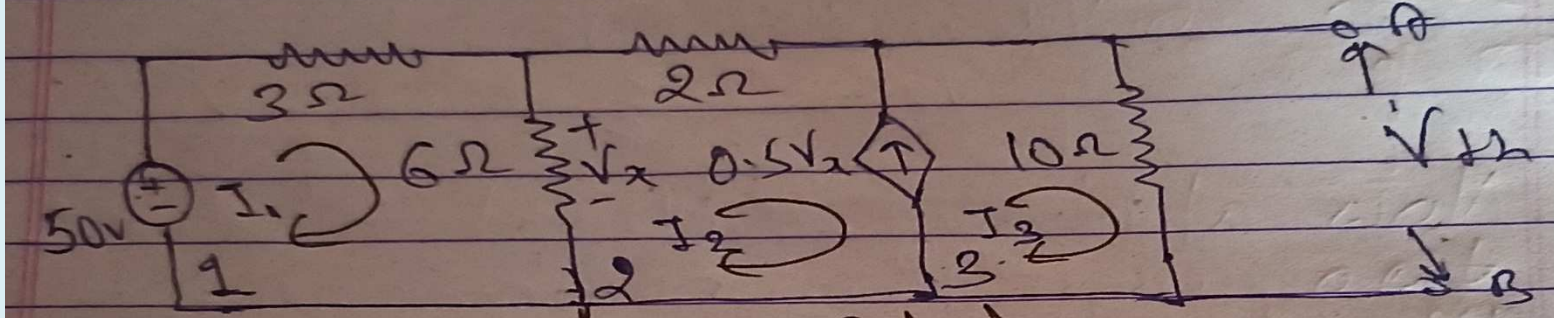

figure B(4)

So V_{th} is voltage across 10Ω resistor.
To find V_{th} we apply KVL in loop 1, 2 & 3 of n/w shown in figure B(4).

Let current I_1, I_2 & I_3 flow in loops 1, 2 & 3 respectively, so applying KVL in loop 1,

$$3I_1 + 6I_1 - 6I_2 - 50 = 0$$

$$9I_1 - 6I_2 = 50 \quad \text{——— (ii)}$$

Circuit Theorem

Now applying KVL in loop 2,

$$2I_2 + 6I_2 - 6I_1 = 0$$

$$8I_2 - 6I_1 = 0$$

or $4I_2 = 3I_1$

putting value of I_1 in eqn (i) we have

$$12I_2 - 6I_2 = 50$$

$$\cancel{6}3I_2 = \cancel{50}\ 25$$

$$I_2 = \frac{25}{3}$$

$$I_1 = \frac{4\times 25}{3\times 3} = \frac{100}{9}$$

Circuit Theorem

$$I_1 = \frac{4 \times 25}{3 \times 3} = \frac{100}{9}$$

Now,

$$0.5\,V_x = I_3 - I_2 \quad \text{—— (iii)}$$

and

$$V_x = 6(I_1 - I_2)$$

$$V_x = 6\left(\frac{100}{9} - \frac{25}{3}\right)$$ putting value of I_1 & I_2

$$V_x = \cancel{6}^{2} \times \frac{25}{\cancel{9}\,3} = \frac{50}{3}$$

Now, putting value of V_x in eqn. (iii) we get

$$0.5 \times \frac{50}{3} = I_3 - \frac{25}{3}$$

$$\Rightarrow I_3 = \frac{50}{6} + \frac{25}{3} = \cancel{\frac{100}{6}}\ \frac{50}{3}$$

Circuit Theorem

$$V_{th} = \cancel{50 \times 10} \quad 10\, I_3$$

$$= 10 \times \frac{50}{3} = \frac{500}{3} = 166.67V$$

Now $$P_{max} = \frac{\left(\frac{500}{3}\right)^2}{4 \times 10}$$

$$= \frac{50\cancel{0} \times \cancel{500}\ 125}{9 \times \cancel{4} \times 1\cancel{0}}$$

$$= \frac{6250}{9} = 694.44 \text{ Watt.}$$

$P_{max} = 694.44$ Watt.