Paper 1, TDC Part-1
Chapter– 4, Circuit Theorems
Lecture - 7

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Circuit Theorem

In today lecture we will see next theorem, which "Maximum Power Transfer Theorem".

This theorem is useful in electronic and communication network, where the aim is to receive or transmit maximum power.

It has crucial significance in the operation transmission line s and antennas.

The application of this theorem is limited to power transmission and distribution networks because here the aim is to achieve high efficiency.

Circuit Theorem

- As applicable to D.C. networks, the maximum power transfer theorem may be stated as follows:-

In a linear DC network the maximum power delivered to the resistive load by the active source only when the restive load is equal to the resistance of the network as viewed from the output (load) terminal.

The network resistance can be obtained as obtained in Thevenins's or Norton's Theorem.

Circuit Theorem

- While for A.C. networks, the maximum power transfer theorem may be stated as follows:-

In a AC network the maximum power delivered to the complex load by the active AC source only when the load impedance is equal to the complex conjugate of the network impedance as viewed from the output (load) terminal.

The impedance of the network can be obtained as obtained in Thevenin's or Norton's Theorem.

Circuit Theorem

Proof of Maximum Power Theorem

Let construct a N/W.

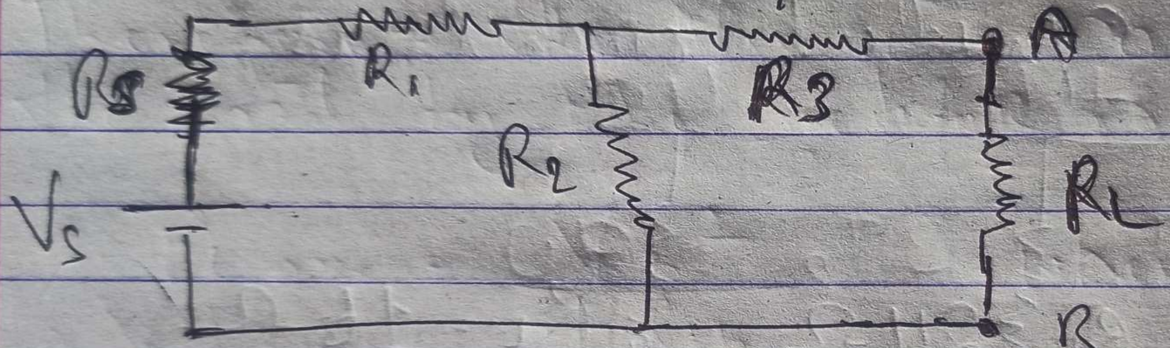
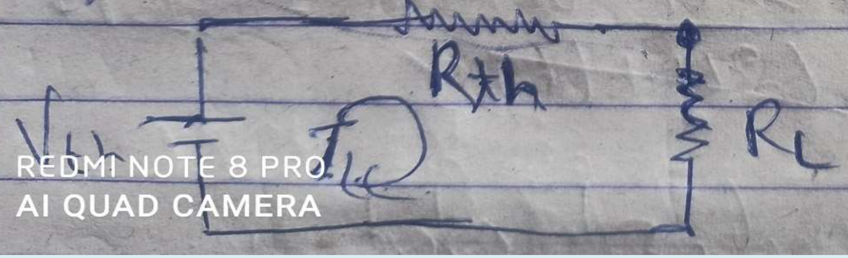
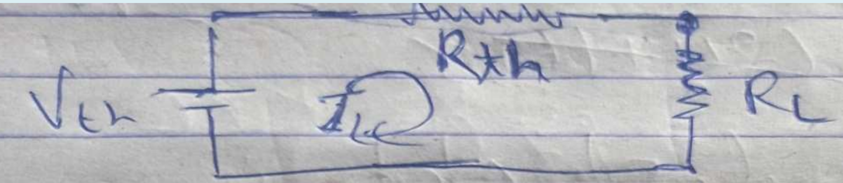


fig 1(a) DC Network

Equivalent ~~Theorem~~ Thevenin's Ckt can be drawn as below



Circuit Theorem



Loop eqn. is :→

$$I_L R_{th} + I_L R_L = V_{th}$$

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

Power consumed by load is

$$P_L = I_L^2 R_L = \frac{V_{th}^2}{(R_{th} + R_L)^2} \times R_L \quad \text{—— (i)}$$

$$I_L^2 R_L$$

For P_L to maximam, differentiate eqn (i) with respect to R_L and equate it with zero.

$$\frac{dP_L}{dR_L} = 0$$

Circuit Theorem

So, $\frac{dP_L}{dR_L} = \frac{d}{dR_L}\left[\frac{V_{th}^2 \times R_L}{(R_{th}+R_L)^2}\right] = 0$

$\Rightarrow V_{th}^2\left[\frac{(R_{th}+R_L)^2 \cdot 1 - R_L \cdot 2(R_{th}+R_L)}{(R_{th}+R_L)^{\cancel{2}\,4}}\right] = 0$

or, $V_{th}^2\left[(R_{th}+R_L)^2 - R_L \cdot 2(R_{th}+R_L)\right] = 0$

or, $\left[(R_{th}+R_L)^2 - R_L \cdot 2(R_{th}+R_L)\right] = 0$

or, $(R_{th}+R_L)^{\cancel{2}} = 2R_L\,\cancel{(R_{th}+R_L)}$

$\Rightarrow R_{th} = 2R_L - R_L$

$R_{th} = R_L$

Circuit Theorem

$\Rightarrow \quad R_{th} = R_L$

So if the value of resistive load is equal to Thevenin's resistance, which is equivalent resistance of N/w looking into the network from source terminal, the the power dissipated across the load will be maximum.

So, $P_{L(max)} = I_L^2 R_L$

$$= \frac{V_{th}^2 \times R_L}{(R_{th} + R_L)^2}$$

$$= \frac{V_{th}^2 \times R_{th}}{(R_{th} + R_{th})^2} \qquad [\because R_L = R_{th}]$$

Circuit Theorem

$$= \frac{V_{th}^2 \times R_{th}}{(2R_{th})^2} = \frac{V_{th}^2 \cancel{R_{th}}}{4R_{th}^{\cancel{2}}}$$

Therefore; the Maximum powe tranfer to the load by the network is,

$$P_{L,max} = \frac{V_{th}^2}{4R_{th}} \quad \text{when } (R_L = R_{th})$$

Q

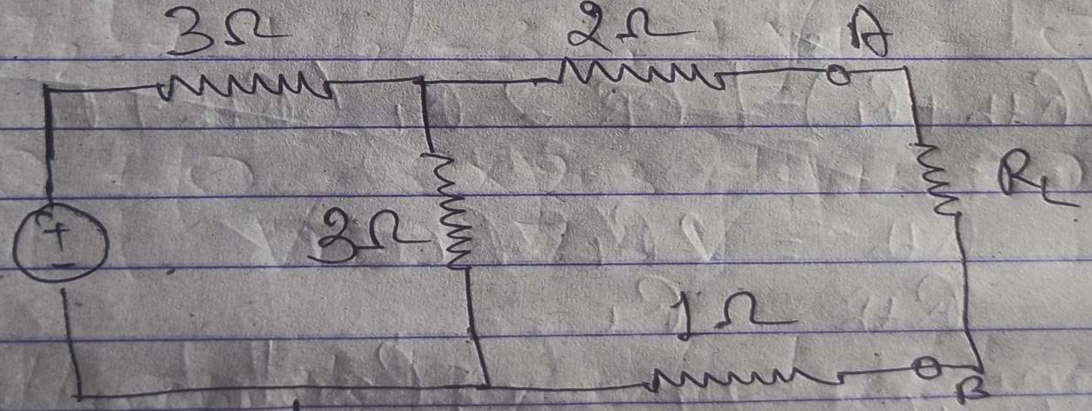


Figure - 1(a)

Circuit Theorem

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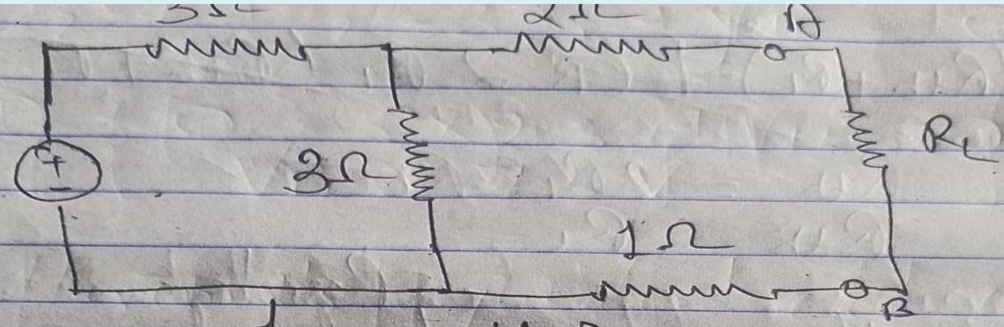


figure - 1(a)

find the value of load R_L such that maximum power is delivered by the n/w to the load. Also find the maximum power and power supplied by source.

Soln. To find the value of load R_L such that maximum power is delivered by the n/w we draw the thevenins equivalent circuit of n/w drawn in figure 1(a)

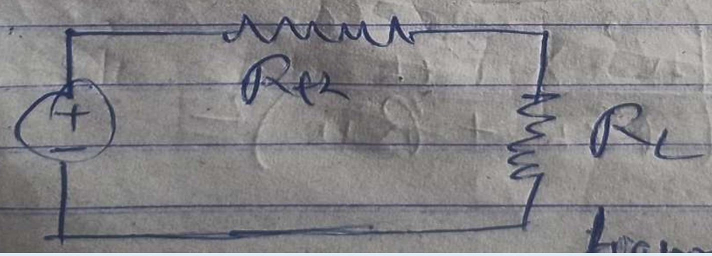


figure 1(b)

Circuit Theorem

To find R_{th} we follow the procedure followed in Thevenin's problem, for that

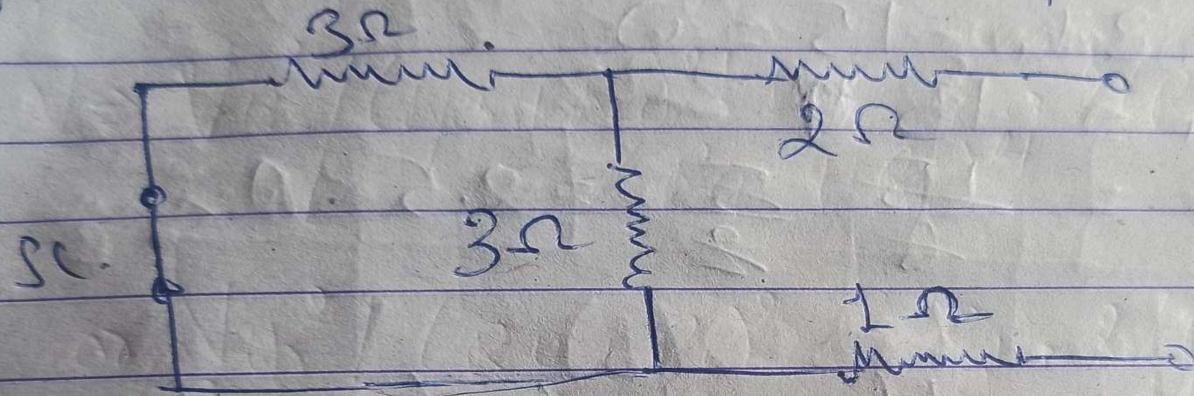


figure 1(c)

$$R_{th} = (3\Omega \,||\, 3\Omega) + 2\,\Omega + 1\,\Omega$$

$$= \left(\frac{\cancel{3}\times\cancel{3}}{\cancel{6}_{2}}\,1.5 + 3\right)\,\Omega$$

$$= 4.5\,\Omega$$

Circuit Theorem

Now to find V_{th} we will again draw the n/w of figure 1(a).

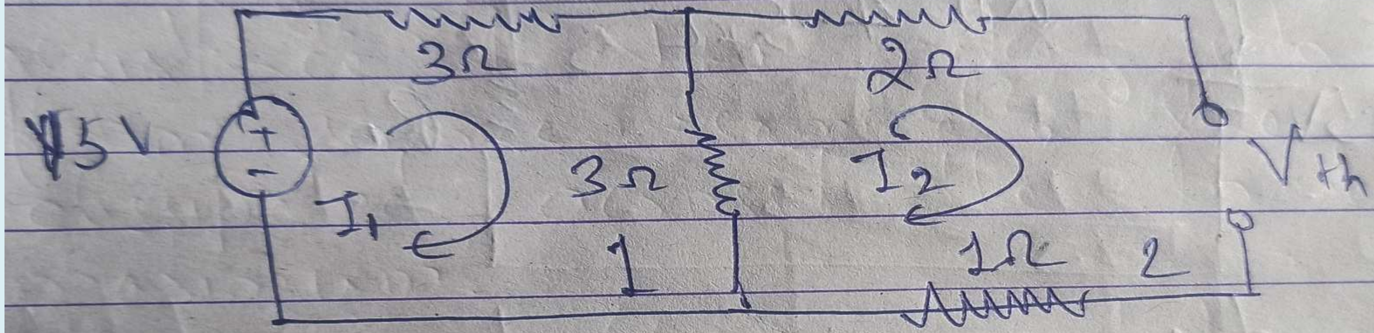


figure 1(d)

Let current I_1 & I_2 flow in loop 1 & 2 respectively.

As loop 2 is open so no current will flow in loop 2

hence value of $I_2 = 0$

Circuit Theorem

In loop 1 applying KVL, we get

$$3I_1 + 3I_1 - 15 = 0$$

$$6I_1 = 15$$

$$I_1 = \frac{\cancel{15}}{\cancel{6}} = \frac{5}{2} = 2.5 \text{ A}$$

For V_{th} we will write ~~loop~~ equation ~~for loop~~

$$2I_2 + 1I_2 - 3I_1 + V_{th} = 0$$

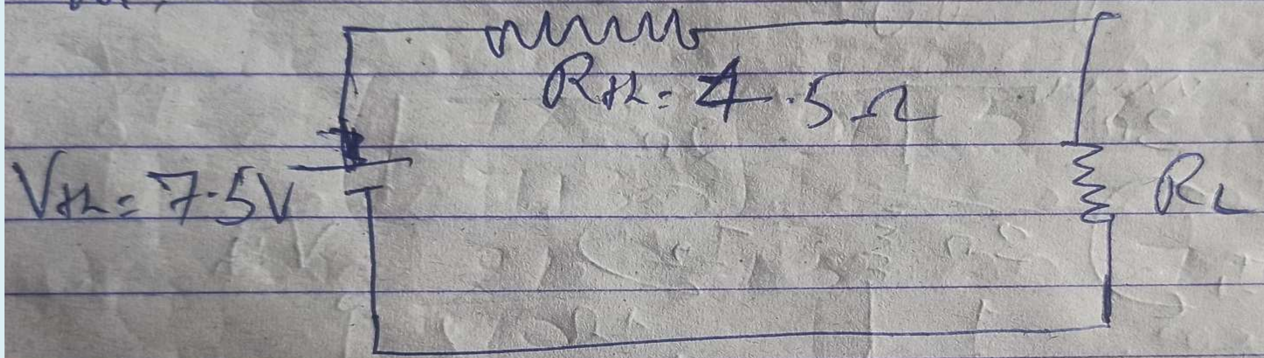
$$0 + 0 - 3\times 2.5 + V_{th} = 0$$

$$V_{th} = 7.5 \text{ V}$$

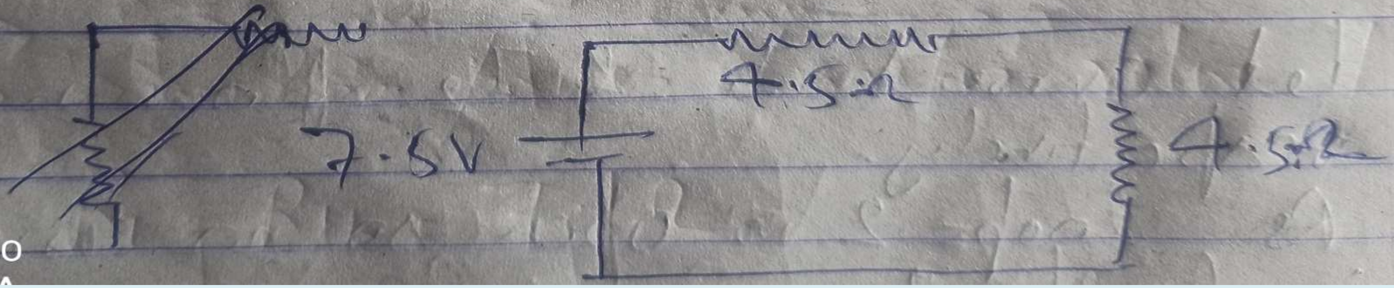
Circuit Theorem

$V_{th} = 7.5\,V$

So Thevenin's equivalent ckt can be drawn as,



So to draw maximum power by load resistance R_L, $R_L = R_{th} = 4.5\,\Omega$



Circuit Theorem

Maximum power drawn by R_L

$$P_L = \frac{V_{th}^2 \times R_{th}}{(R_{th} + R_{th})^2} = \frac{V_{th}^2}{4\,R_{th}}$$

$$= \frac{5\,\cancel{7.5} \times \cancel{7.5}\,2.5}{4 \times \cancel{4.5}\,3} = \frac{12.5}{4}$$

$$= 3.125\ W$$

Power supplied by ~~load~~ source is $= 2 \times P_L$ because same power is developed in R_{th}

So, $P_s = 2 \times 3.125\ W$
$= 6.25\ W$

Circuit Theorem

Q2)

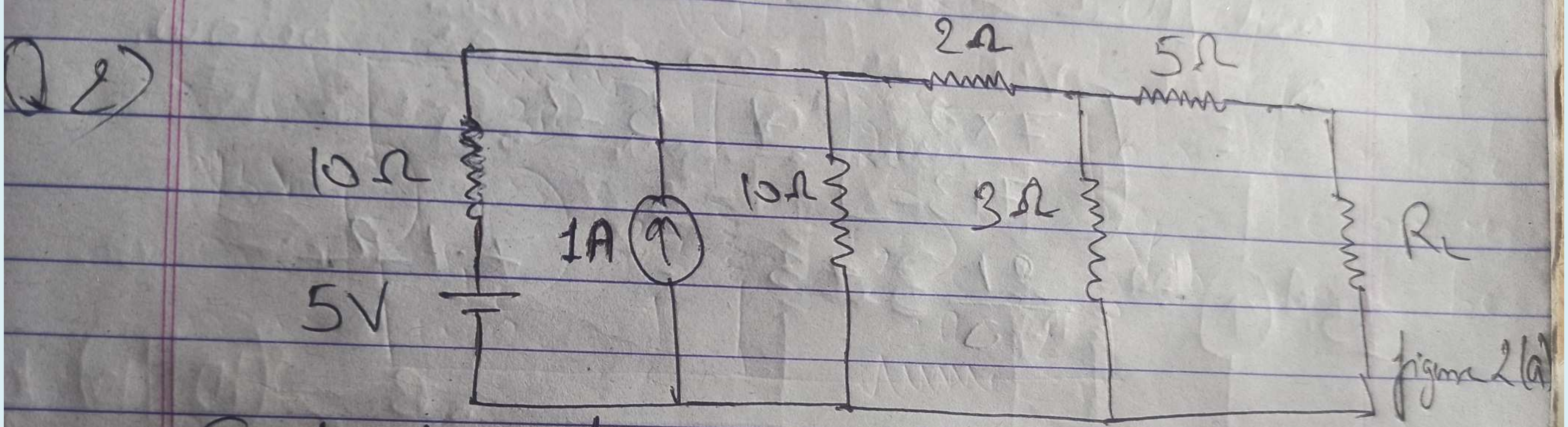


figure 2(a)

Find the value of R_L such that maximum power is transfer by the sources to the n/w. Hence determine the maximum power transfer!

Soln: For maximum power t/f. we known that the value of $R_L = R_{th}$ so for that we find R_{th} & V_{th} by again redrawing the ckt.

Circuit Theorem

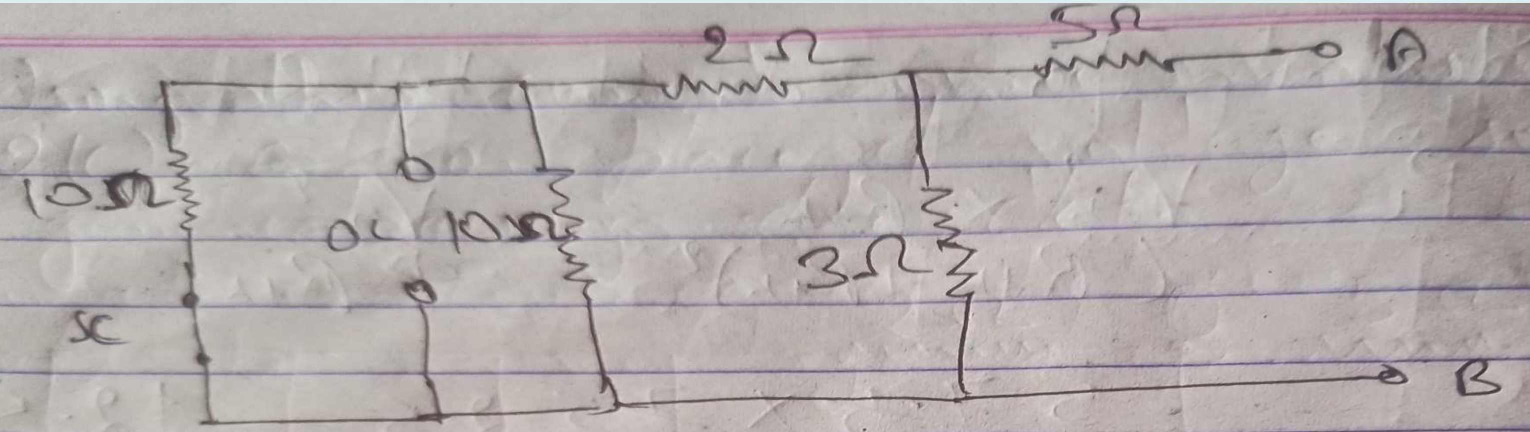


figure 2(b)

$$R_{th} = \left[\left\{(10\,\Omega \parallel 10\,\Omega) + 2\,\Omega\right\} \parallel 3\,\Omega\right] + 5\,\Omega$$

$$= \left[\left\{\left(\frac{\cancel{10\times10}^{\,5}}{\cancel{10+10}}\right) + 2\right\} \parallel 3\,\Omega\right] + 5\,\Omega$$

$$= \left[7\,\Omega \parallel 3\,\Omega\right] + 5\,\Omega$$

Circuit Theorem

$$= [7\Omega \parallel 3\Omega] + 5\Omega$$

$$= \left[\frac{7\times 3}{7+3}\right] + 5\Omega$$

$$= \frac{21}{10} + 5 = 7.1\Omega$$

$$R_{th} = 7.1\Omega$$

So for maximum power transfer value of R_L is

$$R_L = R_{th} = 7.1\Omega$$

Circuit Theorem

Now to find the value of maximum power transfer we have to find V_{th} i.e. open circuit voltage at terminal AB. for that we again draw ckt of figure 2(a) as below.

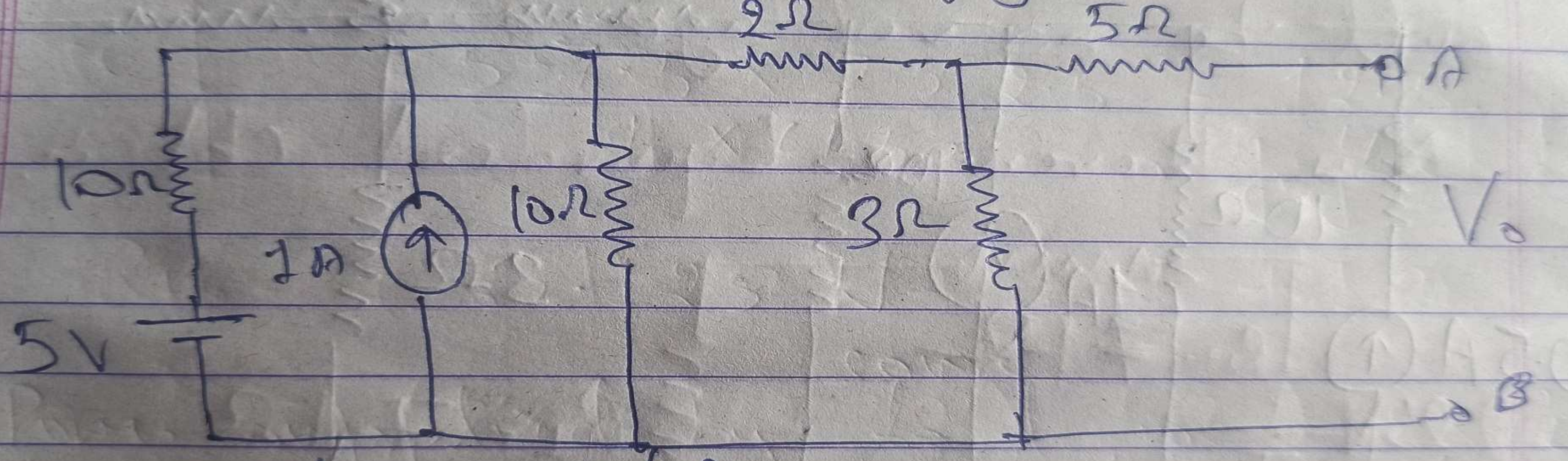


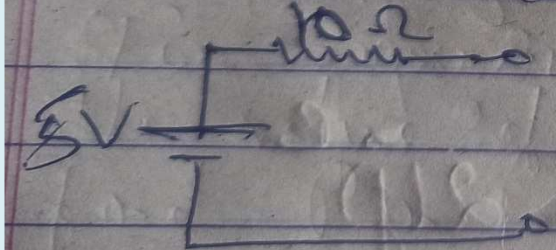
figure 2(c).

To find V_{th} we can use any process like Nodal analysis, source transformation

Circuit Theorem

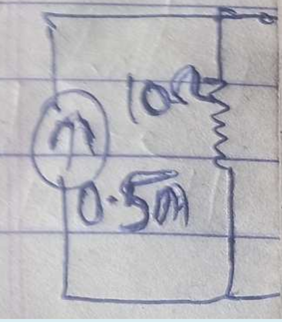
Here we will use source transformation either current source can be transformed to the voltage source or we can transform voltage source into current source.

Let's transform voltage source into the current source. So here we transform 5V voltage source in series with 10Ω resistance to current source with 10Ω ~~par~~ parallel resistance i.e. as below.



equivalent current source is

$$i = \frac{5}{10} = 0.5A$$



Circuit Theorem

Using this current source in n/w of fig 2(C) ~~and~~ in place of voltage source. Again drawing the ckt.

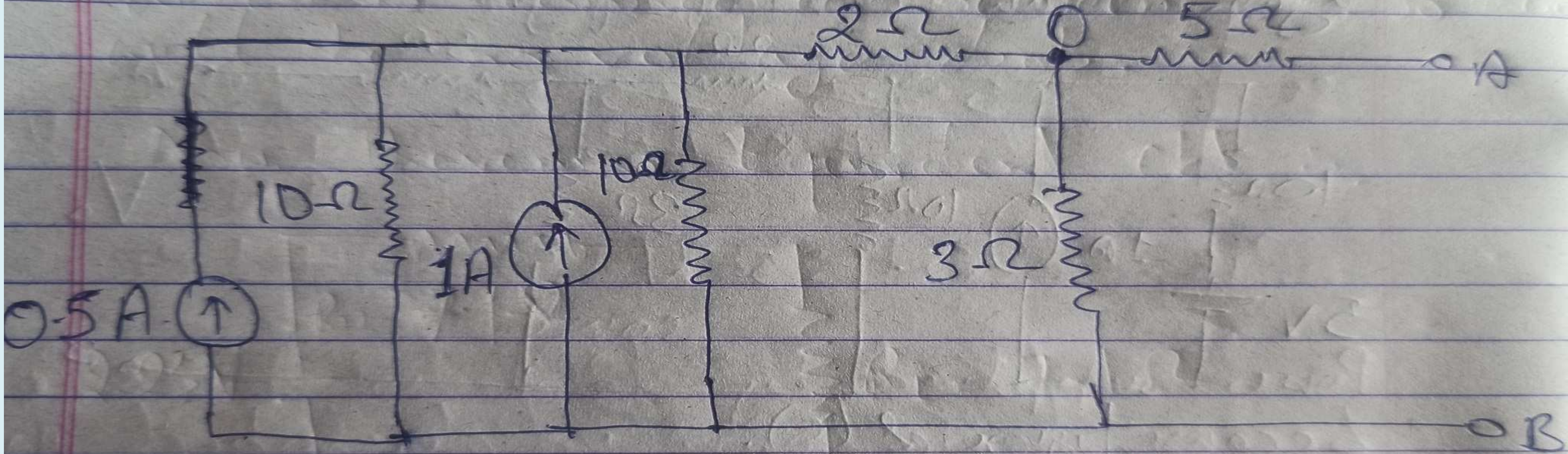


figure 2(d)

The parallel current sources will be added and 10Ω & 10Ω resistors are parallel so, the n/w ~~will~~ can be drawn as below,

Circuit Theorem

The parallel current sources will be added and 10Ω & 10Ω resistors are parallel so, the n/w ~~will~~ can be drawn as below,

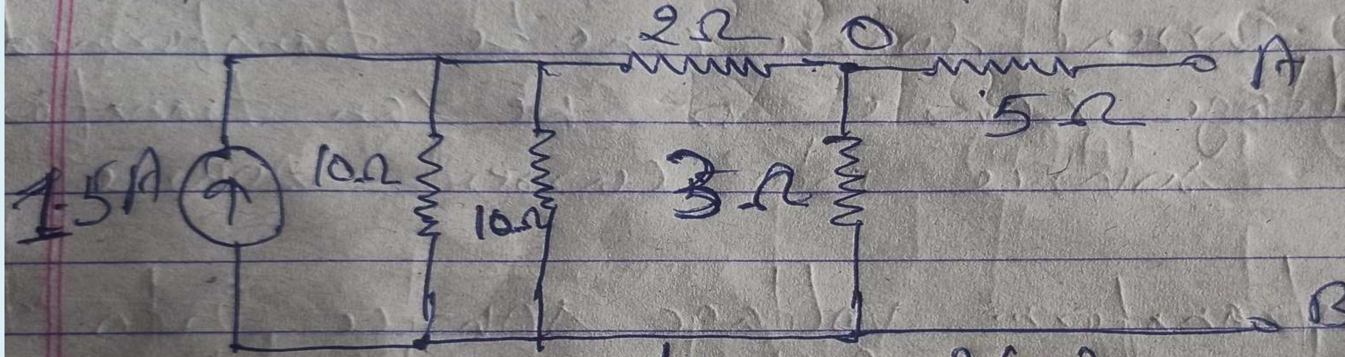


figure 2(e)

≈ or,

$(10 \| 10) = \frac{10}{2} = 5\Omega$

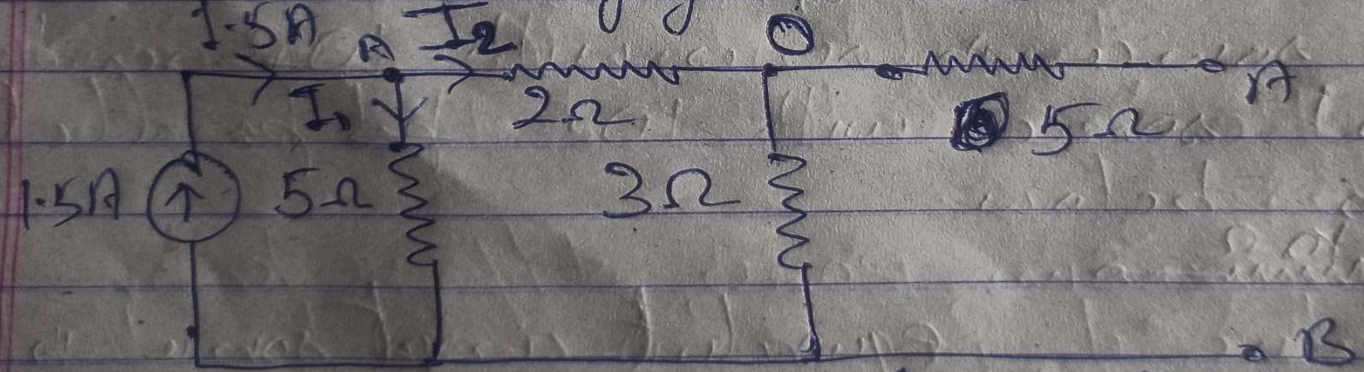


figure 2(f)

Circuit Theorem

Now we can ~~solve~~ find the current through 3Ω resistor. From the figure 2(j) it can be seen that $1.5A$ current will divide at node A and I_1 will flow through 5Ω resistor and I_2 will flow through the series combination of 2Ω and 3Ω resistance. No current will go towards open end containing 5Ω resistor.

So series of 2Ω & 3Ω will result in 5Ω, hence the ~~ckt can be~~ current of $1.5\,A$ will divide between parallel combination of 5Ω & $(2\Omega+3\Omega)$ i.e.

Circuit Theorem

$I_1 + I_2 = 1.5A$; $I_1 = I_2$

$2I_2 = 1.5A$

or, $I_2 = 0.75A.$

This $I_2 = 0.75A$ will flow through 3Ω resistor so, the voltage across 3Ω resistor is,

$$V_{3\Omega} = 3 \times 0.75A = 2.25V.$$

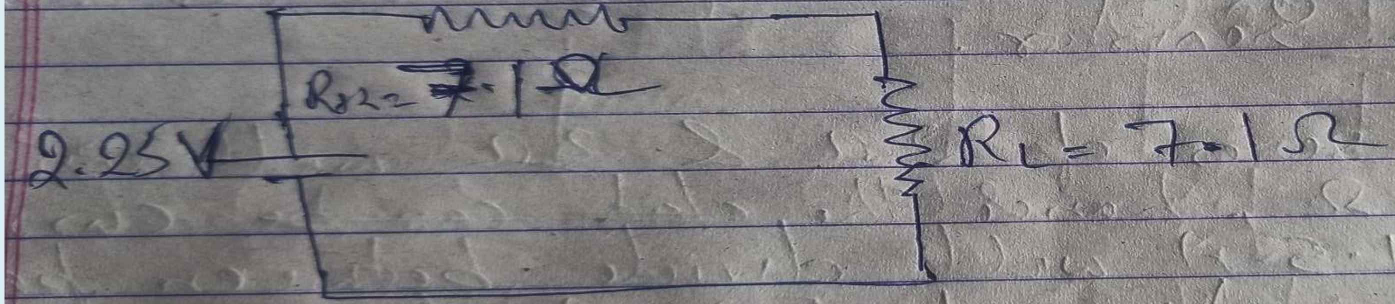
This voltage is ||rl to open terminal AB so,

Circuit Theorem

$V_{AB} = V_{OC} = V_{th} = 2.25\,V$

As there is no current in 5Ω resistor which is connected between OA. So there will be no voltage drop.

Now, Thevenins Equivalent ckt is.



Now $P_{max} = \frac{V_{th}}{4 \times (R_L)} = \frac{V_{th}}{4 \times R_{th}}$

$= \frac{2.25}{4 \times 7.1\Omega} = 0.08\,W.$