03/05/2021 TDC-I

Exp. If the normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle θ with the x-axis, show that its equation is $y\cos\theta - x\sin\theta = a\cos 2\theta$.

Solution From the Eqn of the normal

$$\frac{X-x}{\frac{\partial f}{\partial x}} = \frac{Y-y}{\frac{\partial f}{\partial y}} \quad \text{--- (1)}$$

Let $f(x,y) = x^{2/3} + y^{2/3} = a^{2/3}$

$$\frac{\partial f}{\partial x} = \frac{2}{3} x^{2/3-1} + 0 = 0$$

$$\frac{\partial f}{\partial x} = \frac{2}{3} \cdot \frac{1}{x^{1/3}}$$

similarly $\frac{\partial f}{\partial y} = \frac{2}{3} y^{\frac{2}{3}-1} = \frac{2}{3} \cdot \frac{1}{y^{1/3}}$

$\because$ from Eqn (1) the normal Equation becomes

$$\frac{X-x}{\frac{2}{3} \cdot \frac{1}{x^{1/3}}} = \frac{Y-y}{\frac{2}{3} \cdot \frac{1}{y^{1/3}}}$$

$$Y-y = \frac{x^{1/3}}{y^{1/3}} (X-x) \quad \text{--- (ii)}$$

which we have put in the form

$$y = mx + c$$

since the line (ii), makes an angle θ with x axis. therefore.

$m = \dfrac{x^{1/3}}{y^{1/3}}$ from (11)

$\tan\theta = m$

$\dfrac{\sin\theta}{\cos\theta} = \dfrac{x^{1/3}}{y^{1/3}}$

$\dfrac{x^{1/3}}{\sin\theta} = \dfrac{y^{1/3}}{\cos\theta} = \dfrac{\sqrt{x^{2/3}+y^{2/3}}}{\sqrt{\sin^2\theta+\cos^2\theta}} = \dfrac{\sqrt{a^{2/3}}}{1} = a^{1/3}$

$\Rightarrow x^{1/3} = a^{1/3}\sin\theta \Rightarrow x = a\sin^3\theta$

$y^{1/3} = a^{1/3}\cos\theta \Rightarrow y = a\cos^3\theta$

putting in (11) we get the normal Equation.

$$y - a\cos^3\theta = \frac{a^{1/3}\sin\theta}{a^{1/3}\cos\theta}(x - a\sin^3\theta)$$

$$y\cos\theta - a\cos^4\theta = x\sin\theta - a\sin^4\theta$$

on in terms of x & y

$$y\cos\theta - x\sin\theta = a[\cos^4\theta - \sin^4\theta]$$

$$= a(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$$

$$= a \cdot 1 \cdot \cos 2\theta$$

$$y\cos\theta - x\sin\theta = a\cos 2\theta$$

proved