SEM-II (MPHYCC-6)

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04.05.2020

Unit-3
Lienard-Wiechert potential & Field of Moving charges

An electric charge produces an electric field when it is stationary, however, the moving charges produce both electric and magnetic field i.e electromagnetic field. The relation between these fields is given by

$$\vec{E} = -\nabla\Phi - \frac{\partial \vec{A}}{\partial t} \longrightarrow (1)$$

The first term in eq^n (1) represents a component of the electric field intensity caused by the position of charge even though it be moving & hence Φ is a scalar field. The magnetic induction $\vec{B}$ is a vector quantity & is equal to a curl of a vector $\vec{A}$ (say). This vector $\vec{A}$ is called magnetic vector potential. The time rate of this vector $\vec{A}$ indicates the another component of electric field intensity.

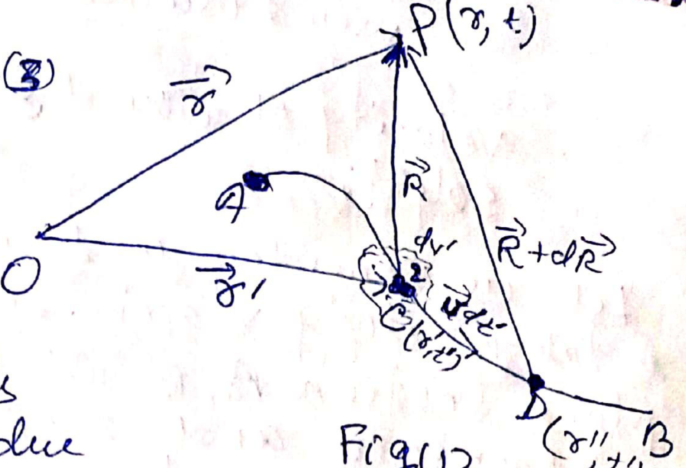
The value of scalar electric potential Φ and vector magnetic potential $\vec{A}$ are obtained by solving the following poisson's equation

$$\nabla^2\Phi = -\rho/\epsilon_0 \quad \& \quad \nabla^2\vec{A} = -\mu_0\vec{J} \rightarrow (2)$$

To solve these equation, let us consider a moving volume charge which is an assembly of constituting charges. Let us consider a constituent charge q moving with velocity v at (r', t'). Here (r', t') are the ~~position~~ co-ordinate of source q and (r, t), the co-ordinate of field point P with respect to an arbitrary origin O as shown in fig (1). Let the signal from q at C reaches at field point P ~~in~~ to with speed u so that time taken will be R/u. Obviously the signal will have been emitted from source q at $(t - R/u)$ sec earlier which is retardation in time.

Thus the retardation time at C will be

$t' = t - R/u \longrightarrow (3)$



Fig(1)

The solution of eq^n (2) in terms of retarded time $t' = t - R/u$ are the retarded potentials $\phi(\vec{r}, t)$ and $A(\vec{r}, t)$ due to volume density of charge $\rho(r', t')$ and current density $\vec{J}(r', t')$. These are

$$\phi(\vec{r}, t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}', t')\,dV'}{R}$$

$$\& \quad A(\vec{r}, t) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}', t')\,dV'}{R} \qquad (4)$$

Since $t' = t - R/u$ is not constant over the volume charge, therefore single dimensional Dirac delta function δ is used to evaluate the integral in eq^n (4). In this way, we obtain

$$\phi(\vec{r}, t) = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{R - \frac{v}{u}\cdot\vec{R}}\right]$$

$$\& \quad A(\vec{r}, t) = \frac{\mu_0 Q}{4\pi}\left[\frac{v}{R - \frac{\vec{v}}{u}\cdot\vec{R}}\right] \qquad (5)$$

For relativistic case $u = c$ (speed of light) then eq^n (5) becomes.

$$\phi(\vec{r}, t) = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{R - \vec{\beta}\cdot\vec{R}}\right]$$

$$\& \quad A(\vec{r}, t) = \frac{\mu_0 Q}{4\pi}\left[\frac{v}{R - \vec{\beta}\cdot\vec{R}}\right] \qquad (6)$$

where, $\beta = v/c$. Eq^n (6) is the relativistic retarded scalar electric potential & vector magnetic potential which depend upon the velocity of moving charge. Thus, the relativistic retarded scalar electric potential and vector magnetic potentials are known as "Lienard Wiechert potential" as investigated by A. Lienard & E. Wiechert for the first time.

Velocity of Sound in An extended ~~Souce~~ Solid:—

Let us consider a solid rod whose length is taken along x-axis. Let us consider two plane section A, B of this rod at a distance x and $x+\delta x$ respectively from the origin as shown in fig (1).

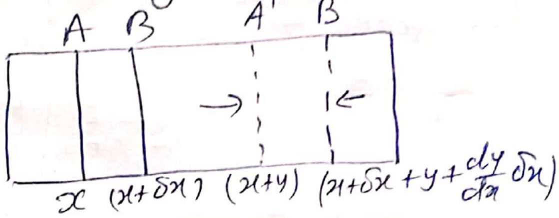


Fig (1)

Let a ~~longitudinal~~ sound wave pass along the axis of the rod. Suppose at some ~~instant~~ particular instant when the wave is passing, the displaced positions of the planes are A' and B' respectively. Let y be the displacement of the plane A. Then the rate of change of displacement with distance is $\frac{dy}{dx}$, so that the displacement of the plane B is $(y+\frac{dy}{dx}.\delta x)$. Thus the new position of the planes, A' and B' are $(x+y)$ and $(x+\delta x+y+\frac{dy}{dx}.\delta x)$ respectively.

The initial length of the slice between the planes A and B.

$$=(x+\delta x)-x=\delta x$$

The final length of the slice between the planes A' and B'

$$=\left(x+\delta x+y+\frac{dy}{dx}.\delta x\right)-(x+y)=\left(\delta x+\frac{dy}{dx}.\delta x\right)$$

$\therefore$ increase in length of the slice

$$=\left(\delta x+\frac{dy}{dx}.\delta x\right)-\delta x=\frac{dy}{dx}.\delta x.$$

$\therefore$ longitudinal strain $= \dfrac{\text{increase in length}}{\text{initial length}}$

$$= \frac{\frac{dy}{dx}\cdot \delta x}{\delta x} = \frac{dy}{dx}.$$

As the slice is under longitudinal strain, there must be a difference in the forces acting on the faces A' and B' due to Hook's law. If Y be the ~~young~~ young's modulus for the material of the rod, and a the area of cross-section of the rod, then the force f acting on the face A' will be given by

$$f = \text{stress} \times \text{area}$$
$$= \text{young's modulus} \times \text{strain} \times \text{area}$$
$$= Y \cdot \frac{dy}{dx} \cdot a$$

The force acting on the face B' of the slice will be in the opposite sense & equal to

$$f + \frac{df}{dx}\cdot \delta x$$
$$= f + \frac{d}{dx}\left(Y \cdot \frac{dy}{dx} \cdot a\right)\cdot \delta x$$
$$= f + Y \cdot a \cdot \frac{d^2y}{dx^2} \cdot \delta x$$

$\therefore$ Resultant force acting on the slice will be

$$f + Y \cdot a \cdot \frac{d^2y}{dx^2}\cdot \delta x - f$$
$$= Y \cdot a \cdot \frac{d^2y}{dx^2} \cdot \delta x$$

If ρ be the density of the material of the rod, the mass of the slice between the planes will be $\rho \cdot a \cdot \delta x$. Then, since force = mass $\times$ acceleration

$\therefore$ The resultant force acting on the slice must be $\rho \cdot a \cdot \delta x \times \dfrac{d^2y}{dt^2}$.

Hence the equation of motion is

$$Ya \frac{d^2y}{dx^2} \cdot \delta x = \rho \cdot a \cdot \delta x \times \frac{d^2y}{dt^2}$$

or, $\dfrac{d^2y}{dt^2} = \dfrac{Y}{\rho} \cdot \dfrac{d^2y}{dx^2}$

Comparing it with the differential equation of wave motion $\dfrac{d^2y}{dt^2} = v^2 \dfrac{d^2y}{dx^2}$, we get $v^2 = \dfrac{Y}{\rho}$ $\therefore v = \sqrt{Y/\rho}$

This gives the velocity of sound wave in a metal rod.

T.D.C. part-II

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Electrodynamics

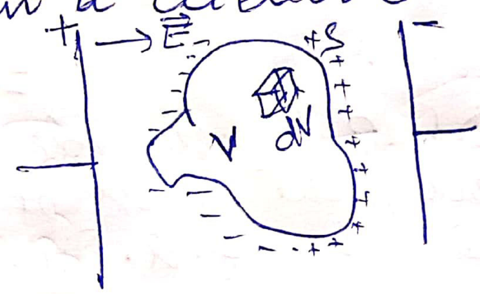
Maxwell's Equation

When charges are in motion, the electric and magnetic fields will be associated with this motion which will have space and time variation. The behaviour of time dependent fields (electric & magnetic) are given by a set of four equations, known as Maxwell's equations of electromagnetic fields. These are

$$
\left.
\begin{aligned}
\vec{\nabla}\cdot\vec{D} &= \rho \quad \cdots\cdots (a)\\
\vec{\nabla}\cdot\vec{B} &= 0 \quad \cdots\cdots (b)\\
\vec{\nabla}\times\vec{E} &= -\frac{\partial \vec{B}}{\partial t} \quad \cdots (c)\\
\vec{\nabla}\times\vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \cdots (d)
\end{aligned}
\right\} \longrightarrow (1)
$$

Derivation of Maxwell's EQs.

First Equation:– Consider a surface S bounding a volume V in a dielectric medium as shown in fig (1). In a dielectric medium, the total charge must include both the free and the polarisation charge. Therefore the total charge density at a point in a small volume element dV would be $(\rho + \rho_p)$, where ρ is the free charge density & ρ_p the charge density due to polarisation



∴ According to Gauss' theorem of electrostatic

$$\oint_S \vec{E}\cdot\vec{dS} = \frac{1}{\epsilon_0}\cdot Q = \frac{1}{\epsilon_0}\oint_V (\rho+\rho_p)dv$$

or $\oint \vec{E}\cdot\vec{dS} = \frac{1}{\epsilon_0}\oint_V (\rho-\vec{\nabla}\cdot\vec{P})dv$ as $\rho_p = -\vec{\nabla}\cdot\vec{P}$

or $\oint (\epsilon_0\vec{E})\cdot\vec{dS} = \oint_V \rho dv - \oint_V \vec{\nabla}\cdot\vec{P}dv$

or $\oint_V (\vec{\nabla}\cdot\epsilon_0\vec{E})dv + \oint(\vec{\nabla}\cdot\vec{P})dv = \oint \rho dv$

(As surface integral is converted into vol^m)

or $\oint_V \vec{\nabla}\cdot(\epsilon_0\vec{E}+\vec{P})dv = \oint \rho dv$

or $\oint_V (\vec{\nabla}\cdot\vec{D})dv = \oint \rho dv$ (∵ Electric displacement $\vec{D} = \epsilon_0\vec{E}+\vec{P}$)

$$\vec{\nabla}\cdot\vec{D} = \rho$$

Thus ~~first law of~~ Maxwell's first Equation is established.

2nd Equation: As the no. of magnetic lines of force entering any arbitrary close surface is exactly the same as leaving it i.e the flux of magnetic induction B across any closed surface is always zero. i.e

$$\int \vec{B}\cdot\vec{dS} = 0$$

Transforming the surface integral int volume integral, we get

$$\int \vec{\nabla}\cdot\vec{B}\,dV = 0.$$

The integrand should vanish for the surface boundary as the volume is arbitrary. ∴ $\vec{\nabla}\cdot\vec{B} = 0$ Proved

Third Equation

According to Faraday law of electromagnetic induction, the e.m.f induced in a closed loop is given by

$$e = -\frac{\partial \phi}{\partial t} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$\because$ the flux $\phi = \int_S \vec{B} \cdot d\vec{S}$ where S is any surface.

The e.m.f e is also defined as work done in carrying a unit charge completely around the loop. Thus

$$e = \oint \vec{E} \cdot d\vec{l}$$

$\vec{E}$ is the intensity of the field associated with induced emf

$\therefore$ on equating above equation, we have

$$\oint \vec{E} \cdot \vec{dl} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot dS$$

or, $$\int_S (\vec{\nabla} \times \vec{E}) \cdot d\vec{S} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$ (using Stokes theorem)

As S is arbitrary, Therefore we have $$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

This is Maxwell's third Equation of electromagnetic field.

~~Proof~~ 4th Eqn :-

According to Ampere's law in the circuital form, the work done in carrying a unit magnetic pole once around a closed arbitrary path linked with the current is expressed as

$$\oint \vec{H} \cdot \vec{dl} = I = \int \vec{J} \cdot \vec{dS}$$

where the integral on the right is taken over the surface through which the charge flow corresponding to the current I takes place.

On changing the line integral into surface integral by Stoke's theorem.

$$\int_S \mathrm{curl}\, \vec{H} \cdot \vec{dS} = \int \vec{J} \cdot \vec{dS}$$

$$\therefore \mathrm{Curl}\, \vec{H} = \vec{J} \longrightarrow ②$$

Taking div. on both sides, so we have.

$$\mathrm{div} \cdot \mathrm{Curl}\, \vec{H} = \vec{\nabla} \cdot \vec{J}$$

$$\text{or,} \quad \vec{\nabla} \cdot \vec{J} = 0 \longrightarrow ③$$ (as div. of curl of a vector is zero)

Eqn ③ contradicts the equation of continuity i.e $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$. Therefore Maxwell realised that the definition of the total current density is incomplete and suggested to add something to $\vec{J}$.

$$\text{i.e} \quad \mathrm{Curl}\, \vec{H} = (\vec{J} + \vec{J}') \longrightarrow ④$$

Taking divergent on both sides, we have

$$\vec{\nabla} \cdot (\vec{J} + \vec{J}') = \mathrm{div}\, \mathrm{Curl}\, \vec{H} = 0$$

$$\text{or} \quad \vec{\nabla} \cdot \vec{J}' = -\vec{\nabla} \cdot \vec{J} = \frac{\partial \rho}{\partial t}$$

or $\vec{\nabla}\cdot\vec{J}' = +\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{D})$ (from Maxwells first Eqn)

or $\vec{\nabla}\cdot\vec{J}' = +\vec{\nabla}\cdot\frac{\partial\vec{D}}{\partial t}$

or $\vec{J}' = \frac{\partial\vec{D}}{\partial t}$

$\therefore$ Eqn (4) becomes:

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$$

This is the Maxwell's fourth equation of electromagnetic field. From these equation we conclude that the source of electric field is the changing magnetic field and vice-versa. Also, both the electric field and magnetic field is curly or circular in nature & the their field directions are perpendicular to each other.

T.D.C. Part III
Electronics

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TRANSISTOR

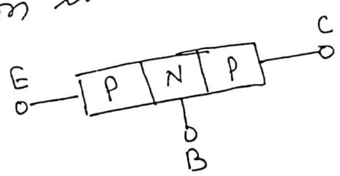
Question:- Give the principle and working of BJT. Explain the mechanism of current flow in P.N.P and N.P.N transistor. Compare the performance of BJT and FET.

ANS

Construction: ~~Principle~~:- The transistor is a solid having three terminals ~~and is an essential~~ state device & two junction device. It is formed by sandwiching one type of semi-conductor material between two layers of the other type. Accordingly, there are two types of transistor;

1. N-P-N transistor
2. P-N-P transistor

When a layer of P type material is sandwiched between two layers of N-type material, the transistor is known as N-P-N transistor as shown in fig (1a). Similarly, when a layer of N-type material is sandwiched between two layers of P-type material, the transistor is known as P-N-P transistor as shown in fig (1b).



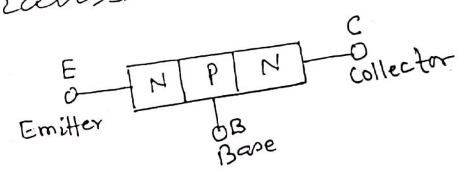


fig (1b).

fig (1a)

The three terminals in the transistor are as under

Emitter:- This forms the left hand section or region of the transistor. The main function of this region is to supply majority charge carriers to the base & hence it is more heavily doped in comparison to other regions.

Base:- The middle section of the transistor is known as "base". This is very lightly doped and is very thin (10^{-6} m) as compared to either emitter or collector so that it may pass most of the injected charge carriers to the collector.

Collector:- The right hand section of the transistor is called collector. The main function of the collector is to collect majority charge carriers through the base. This is moderately doped. As collector has to dissipate much greater power, the collector region is made physically larger than emitter. Hence collector and emitter are not interchangeable.

Principle:- A transistor is just like two diodes. The junction between emitter and base ~~and~~ called emitter diode, and junction between collector and base known as collector diodes. There will be two depletion regions at the two junctions as shown in fig (2)

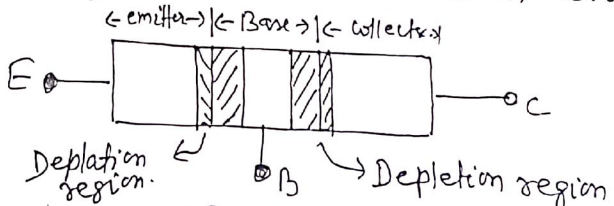


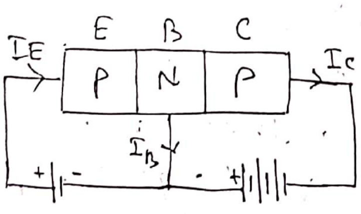
Fig (2)

The width of the two depletion regions will be different because the regions are doped at different levels. depletion region penetrates more ~~depply~~ deeply into lightly doped side so, the depletion region or layer formed at collector junction is larger than that depletion layer formed at emitter junction. Therefore, resistance of the emitter diode is smaller than that of collector diode. In a transistor a weak signal is introduced in low resistance circuit and the output is taken from the high resistance circuit. so a transistor transfers a signal from low resistance to high resistance. The prefix 'trans' means the signal transfer property of the device while 'istor' classifies as a solid element in the same general family with resistors.

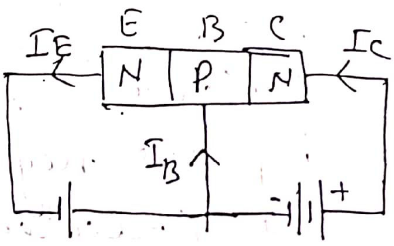
Working:- There are four possible ways of biasing a transistor. These are called as modes of operation of a transistor. These are listed below.

Case	Emitter-base junction	Collector-Base Jn	Region of op.	Act as
I	Forward-biased	Reverse bias	Active	Amplifier
II	Forward-biased	Forward bias	Saturation	closed switch
III	Reverse-biased	Reverse bias	Cut-off	open switch
IV	Reverse-biased	Forward bised	Inverted	poor action.

From the above list, it is clear that for best performance of the transistor emitter-base junction is always forward biased and collector-base junction is reversed biased. This is why the emitter current enters into the base in case of PNP transistor and out come ~~through~~ the ~~the~~ from the base into emitter in case of NPN transistor. The arrow sign on the head of emitter terminals shows the conventional direction of current. The symbolic representation of the transistor is shown in fig (3){(a) & (b)}.



-PNP-



(a) (b)

fig(3)