$$dp_i = \frac{1}{2}\sqrt{2mkT}\, x^{-1/2}\, dx.$$

Then,
$$Q_1 = \frac{4\pi V}{h^3} \cdot \frac{1}{2}(2mkT)^{3/2} \int_0^\infty e^{-x} x^{1/2}\, dx$$

$$= \frac{4\pi V}{h^3} \cdot \frac{1}{2}(2mkT)^{3/2} \cdot \frac{1}{2}\sqrt{\pi}.$$

$$= V\left(\frac{2\pi mkT}{h^2}\right)^{3/2}. \quad \text{——— (6)}$$

Substituting this value in (3), the partition function for an ideal gas is,

$$Q_N = \frac{1}{N!}\left[V\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right]^N \quad \text{——— (7)}$$

Then the free energy of an ideal gas is,

$$A = -kT \log Q_N.$$

$$= -NkT \log\left[V\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right] + kT \log N!$$

Using sterling formula.

$$\log N! = N \log N - N.$$

we get,

$$A = -NkT \log\left[V\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right] + NkT \log N - NkT.$$

$$= -NkT \log\left[\frac{V}{N}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right] - NkT.$$

$$= -NkT \log\left(\frac{V}{N}\right) - \frac{3}{2} NkT \log(kT) - \frac{3}{2} NkT \log\left(\frac{2\pi m}{h^2}\right) - NkT \quad \text{——— (8)}$$

Then the pressure of the gas is,

$$P = -\left(\frac{\partial A}{\partial V}\right) = \frac{NkT}{V}.$$

or, $PV = NkT$ ——— (9)

which is the equation of state for an ideal gas.

The entropy of an ideal gas is,

$$S = -\left(\frac{\partial A}{\partial T}\right)$$

$$= Nk\log\left(\frac{V}{N}\right) + \frac{3}{2}Nk\log(kT) + \frac{3}{2}Nk + \frac{3}{2}Nk\log\left(\frac{2\pi m}{h^2}\right) + Nk.$$

$$= Nk\log\left[\frac{V}{N}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right] + \frac{5}{2}Nk$$

or, $$\frac{S}{k} = N\log\left[\frac{V}{N}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right] + \frac{5}{2}N. \quad \text{—— (10)}$$

which is known as the Sackur-Tetrode equation.

The energy is,

$$U = A + ST.$$

$$= \frac{3}{2}NkT \quad \text{—— (11)}$$

and finally the specific heat at constant volume is,

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{3}{2}Nk.$$

which is independent of temperature.

—— x ——

The pressure of an ideal gas is $P = -\left(\frac{\partial A}{\partial V}\right)_{N,T}$

$$= \frac{NkT}{V}$$

or, $\underline{PV = NkT}$.

2nd ~~Stat.~~ Ensemble → Phase space of particle = configurational space + momentum space.

Volume element in phase space $= dq\,dp$.

$$= dq_1 dp_1, dq_2 dp_2 \ldots dq_n dp_n$$

$$\langle f \rangle = \bar{f} = |\psi^*|\psi\,dq = \int f\rho\,dq, \quad \langle f \rangle = \int f\rho\,dq\,dp$$