M.sc. (Phy), 2nd Sem.

Phase-space → Let us consider a system of N particles moving according to the laws of classical mechanics. A state of the system is spacified by means of 3N position co-ordinates $q_1, q_2, q_3 \dots q_{3N}$ and 3N momentum co-ordinates $p_1, p_2 \dots p_{3N}$. The 6N dimensional space is called phase-space of the system. A point represented by (q_i, p_i) where $i = 1, 2, \dots 3N$ represents a particular state of a system in phase space. The point (q_i, p_i) is called the phase-point or representative point. If the system is in gaseous condition the phase space is also refered to as the Γ-space and the ~~ph~~ phase point is as G-point.

Sometimes 3N position components are seperately called the configurational space and 3N of momentum components as the momentum space. The phase space is thus the sum of configurational space and momentum space. The phase space is a classical concept.

Statistical Ensemble → Gibbs introduced the concept of ensemble for calcalating a average values of physical quantities. A collection of particles is called a system and a collection of ~~identi~~ identical systems is called ensemble provided each system must have the same Hamiltonian and

and they are non-interacting.

For example if a gas with N particles is contained in a box, the particles can be distributed in N! ways in the space. Each arrangement of particles is refered to as a system and collection of N! such system is an ensemble. This is geometrically represented by a distribution of representative points in Γ-space. This is usually a continuous distribution.

In fact an ensemble is a mental picture introduced to replace time average over a single system by an ensemble average.

Statistical distribution function → Let us calculate the ensemble average of a physical quantities.

This is expressed in terms of the probability density $\rho(q, p, t)$ in the phase-space. The function ρ is called the statiscal distribution function or simply distribution function.

This probability dw of the phase points in the volume element $dq\,dp$ arround the phase point (q, p) of the phase space is given by.

$$dw = \rho(q, p, t)\, dq\, dp \quad \text{—} \quad ①.$$

This is the number of phase points in the volume element

$dq\,dp$ at any time t. This function ~~[illegible]~~ satisfies the normalization condition

$$\int \rho(q,p,t)\,dq\,dp = 1 \quad \text{— ②}$$

Integration in ② extends over whole of the phase space.

Ensemble average → The ensemble average $\langle f \rangle$ of a physical quantity $f(q,p)$ ~~in~~ is defined as

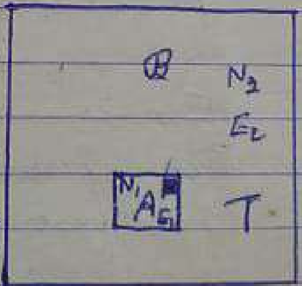
$$\langle f \rangle = \int f(q,p)\,\rho(q,p,t)\,dq\,dp \quad \text{— ③}$$

In general the ensemble average $\langle f \rangle$ may be a function of time. However if ρ does not depend on time explicity

$$\frac{\partial \rho}{\partial t} = 0.$$

Then $\langle f \rangle$ will be independent of time. Such an ensemble is said to be stationary and represents a system in equilibrium.

Gibbs Canonical distribution →



Let us consider an isothermal system i.e. a system in equilibrium with a heat reservoir (thermostat) having a ~~heat~~ common temp T.

The composite system is a large closed system which has the constant energy E. Let A be a system and B the thermostate. Let the system and thermo-

-state consist of N_1 and N_2 particles, respectively, having the energies E_1 and E_2. then,

$$E_1 + E_2 = E = \text{a constant}$$
$$N_1 + N_2 = N = \text{a constant}.$$

Since the thermostate is supposed to be much larger than the system.

$$N_1 << N_2.$$
$$E_1 << E_2.$$

The composite system is considered as adiabatically isolated. Hence the micro cannonical distribution is applicable to ~~sch~~ such a system. Then the probability dW that the system and the reservoir are in the volume element $d\Gamma_1$ and $d\Gamma_2$ respectively is given by,

$$dW = c\,\delta\,(E_1 + E_2 - E_0)\,d\Gamma_1\,d\Gamma_2 \quad \text{—①}$$

where c is a constant and E_0 is giv- given value of the energy of the closed system which is equal to E_1 and E_2.

Let us find out the probability P_n that the system A is in a definite quantum state with energy E_n. In the case $E_1 = E_n$ and $d\Gamma_1 = 1$. The probability P_n can be obtained by integrating ① over $d\Gamma_2$. Thus.

$$P_n = c\int \delta(E_n + E_2 - E_0)\,d\Gamma_2.$$
$$= c\int \delta(E_2 - [E_0 - E_n])\,d\Gamma_2 \quad \text{—②}$$

Let $\Gamma_2(E_2)$ is the number of quantum states of the thermostate and is a function of E_2 Then,

$$\Delta\Gamma_2 = \frac{\partial\Gamma_2}{\partial E_2}\Delta E_2$$

$$\text{or, } \frac{\partial\Gamma_2}{\partial E_2} = \frac{\Delta\Gamma_2}{\Delta E_2}$$

But the entropy of the thermostate is:

$$S_2(E_2) = k \log \Delta\Gamma_2$$

$$\text{or, } \Delta\Gamma_2 = e^{S_2(E_2)/k}$$

$$\therefore \frac{\partial\Gamma_2}{\partial E_2} = \frac{e^{S_2(E_2)/k}}{\Delta E_2} \quad \text{③}$$

Now

$$d\Gamma_2 = \frac{\partial\Gamma_2}{\partial E_2} dE_2$$

$$= \frac{e^{S_2(E_2)/k}}{\Delta E_2} dE_2 \quad \text{④}$$

Substituting this value in ② we get,

$$\rho_n = c\int \delta(E_2 - [E_0 - E_n]) \left[\frac{e^{S_2(E_2)/k}}{\Delta E_2}\right] dE_2 \quad \text{⑤}$$

Using the property of the delta function

$$\int \delta(x-a) f(x)\, dx = f(a).$$

we obtain the result,

$$\rho_n = c\,\frac{e^{S_2(E_0-E_n)/k}}{\Delta(E_0-E_n)} \quad \text{⑥}$$

Since the system A is small, its energy E_n is small in comparison with E_0. Variation of ΔE is negligible when E varies slightly, so we can write -

$$\Delta(E_0 - E_n) \simeq \Delta E_0.$$

Next we expand $S_2(E_0 - E_n)$ about E_0.

$$S_2(E_0 - E_n) = S_2(E_0) - E_n \frac{\delta S_2(E_0)}{\partial E_0}$$

$$= S_2(E_0) - \frac{E_n}{T}.$$

where $\frac{\partial E_0}{\partial S_2} \simeq T$.

Thus we have an expression for ρ_n as,

$$\rho_n = c\, \frac{e^{S_2(E_0)/k}}{\Delta E_0}\, e^{-E_n/kT}$$

$$= A\, e^{-E_n/kT} \quad \text{———} ⑦$$

where $A = c\, \frac{e^{S_2(E_0)/k}}{\Delta E_0}$ is a normalisation constant and is given by the normalisation condition.

$$\sum_n \rho_n = 1.$$

Thus $$1/A = \sum_n e^{-E_n/kT}. \quad \text{———} ⑧$$

⑦ is known as Gibbs cannonical distribution or simply Gibbs distribution for a closed system in quantum statistics.

In classical statistics a similar expression ~~for~~ for Gibbs distribution function in phase space is,

$$\rho(q,p) = A e^{-E(q,p)/kT} \quad \text{———} ⑨.$$

where $E(q,p)$ is the energy of the system as a function of its co-ordinates q and momenta p. The normalization constant A is given by the condition

$$\int \rho(q,p)\, dq\, dp = 1.$$

or, $$\frac{1}{A} = \int e^{-E(q,p)/kT}\, dq\, dp \quad \text{———} ⑩$$

PAPA (MAA)

B.Sc. Ist Part Phy (H.)

Sh. Q. → Explain Inertial and Non-Inertial frame of reference. Discuss frame of reference in use.

An inertial system is defined as a frame of reference in which the law of inertia - Newton's first law holds. In such a system, which we may also describe as an unaccelerated system, a body that is acted on by zero net external force will move with a constant velocity. Newton assumed that a frame of reference fixed with respect to the stars is an inertial system. A rocket ship drifting in outer space, without spinning and with its engines cut off, provides an ideal inertial system. Frames accelerating with respect to such a system are non-inertial.

In practice, we may often neglect the small (acceleration) effects due to the rotation and the orbital motion of the earth and to solar motion. Thus, we may regard any set of axes fixed on the earth as forming (approximately) an inertial coordinate system. Likewise, any set of axes moving at uniform velocity with respect to the earth, as in a train, ship, or airplane, will be (nearly) inertial because motion at uniform velocity does not introduce acceleration. However, a system of axes ~~fixed on the earth~~ which accelerates with respect to the earth, such as one fixed to a spinning merry-go-round or to an accelerating car, is not an inertial system. A particle acted upon by zero net external force will not move in a straight line with constant speed according to an observer in such non-inertial systems.

At the present time the inertial character of the heliocentric reference frame (fixed to the centre of the sun and stationary stars) is confirmed by the whole totality of experimental facts.

There are three frames of reference that are predominatntly preferred by physicists:

(i) Laboratory frame (or simply L-frame, or Lab. frame): –

In this frame all the energies, momenta and directions of the particles are taken such as they are recorded in the instrument at rest in the laboratory. For example, in the case of microparticles, in the chamber or on the photographic film.

(ii) Co-moving frame (or simply, COframe) :-

This frame travels along with the moving particle. It is convenient particularly in the decay process for the decay laws can be readily conceived of. It is also called the proper frame. Obviously, in its proper frame, a particle is always at rest.

(iii) The C-frame (frame of the Centre of inertia) :-

The reference frame rigidly fixed to the centre of inertia of a given system of particles and translating with respect to inertial frames is referred to as the frame of the centre of inertia or, briefly, the C-frame. The distinctive feature of the C-frame is that the total momentum of the system of particles is equal to zero.

In other words, any system of particles as a whole is at rest in its c-frame. It is also called zero momentum frame (ZMF).

Ehrenfest's Theorem & its proof :-

Ehrenfest was the German scientist who established his theorem in the year 1927. According to the theorem the motion of the expectation values of the co-ordinates & momenta of a dynamical system are governed by the equations which are similar to the corresponding equations in Newtonian mechanics.

Next, we will outline the proof for one particle system constrained to move in one dimension only.

Proof :- Let us consider the two operators $\hat{A}$ and $\hat{B}$, then according to Prof. Heisenberg

$$[\hat{A}, \hat{B}] = i\hbar \frac{\partial}{\partial t} \hat{A} .$$

If $i\hbar \frac{\partial}{\partial t} \hat{A} = 0$, then $\hat{A}$ = conserved = constant

i.e. time-independent.

Next,

$$i\hbar \frac{\partial}{\partial t} \hat{A} = [\hat{A}, \hat{H}]$$

symbols have their usual meaning.

or $i\hbar \frac{\partial}{\partial t} \hat{r} = [\hat{r}, \hat{H}]$, symbols have their usual meaning.

or $$i\hbar \frac{\partial}{\partial t} \hat{r} = \left[\hat{r}, \frac{p^2}{2m} \right] \quad \text{———— (1)}$$

Now, $$i\hbar \frac{\partial}{\partial t} \hat{r} = \frac{r p^2}{2m} - \frac{p^2}{2m} r$$

$$= \frac{1}{2m} \left(r p^2 - p^2 r \right)$$

$\because [A, B] = AB - BA$

$$= \frac{1}{2m} \left[r, p^2 \right]$$

$$\Rightarrow i\hbar \frac{\partial}{\partial t} \hat{r} = \frac{1}{2m} \left\{ [r, p] p + p [r, p] \right\} \quad \text{———— (2)}$$

$\because [r, p^2] = [r, p] p + p [r, p]$

We know that

$$[x, p_x] = x p_x - p_x x$$

$$= x \left(-i\hbar \frac{\partial}{\partial x} \right) - \left(-i\hbar \frac{\partial}{\partial x} \right) x$$

$$\therefore [x, p_x] = i\hbar \quad \text{———— (3)}$$

With the help of (3), we write (2) as

$$i\hbar \frac{\partial}{\partial t} \hat{r} = \frac{1}{2m} \left\{ i\hbar p + p \, i\hbar \right\}$$

$$\therefore\ i\hbar \frac{\partial}{\partial t}\hat{r} = \frac{i\hbar}{2m}\cdot 2p$$

$$\Rightarrow\ \dot{\hat{r}} = \frac{p}{m} \qquad \text{———} \quad (4)$$

Similarly, for further extension of the proof, we have

$$i\hbar \frac{\partial}{\partial t}\hat{A} = [\hat{A}, \hat{H}]$$

If $i\hbar \frac{\partial}{\partial t}\hat{p} = [\hat{p}, \hat{H}] = 0$, then $\hat{p}$ = conserved = constant.

Next, $i\hbar \frac{\partial}{\partial t}\hat{p} = \left[p, \frac{p^2}{2m} + V\right]$

$$= p\left(\frac{p^2}{2m} + V\right) - \left(\frac{p^2}{2m} + V\right)p$$

$$= \frac{p^3}{2m} + pV - \frac{p^3}{2m} + Vp$$

$$= [p, V]$$

$$= pV - Vp$$

$$= (-i\hbar\nabla)V - V(-i\hbar\nabla)$$

$$= -i\hbar\nabla V$$

$$\Rightarrow\ i\hbar\frac{d}{dt}p = i\hbar\dot{p} = -i\hbar\nabla V$$

$$\therefore\ \dot{p} = F \qquad \text{——} \quad (5) \qquad \Big|\ \because F = m\ddot{r} = -\nabla V$$

Electrostatics B.Sc. 2nd Part Phy.(H)

Ques: What is Clausius - Mossotti relation ?

Under the action of electric field E_m, molecule displays an electric moment p in the direction of the field which is dependent upon the field strength. Therefore we write

$$p = \alpha E_m \qquad \text{——— (1)}$$

Where α = molecular polarizability. If there are n molecules per unit volume of the dielectric, then the polarization (electric moment/volume) is given by

$$P = np = n\alpha E_m$$
$$= n\alpha \left(E + \frac{P}{3\epsilon_0}\right) \qquad \text{——— (2)}$$

ϵ_0 = permitivity in vacuum.

This equation may be rewritten in terms of dielectric constant K. As we know,

$$P = (K-1)\,\epsilon_0\, E \qquad \text{——— (3)}$$

We write (2) as

$$\frac{n\alpha}{3\epsilon_0} = \frac{(K-1)}{(K+2)}$$

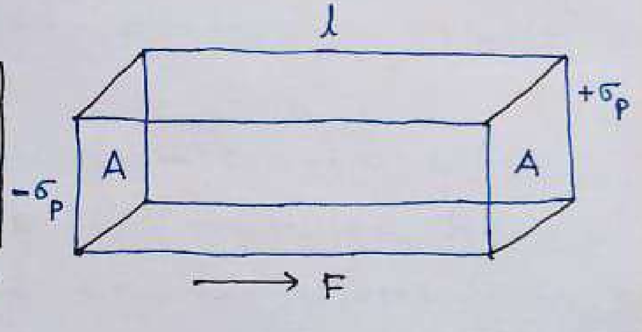
i.e. $$\alpha = \frac{3\epsilon_0}{n}\,\frac{(K-1)}{(K+2)} \qquad \text{——— (4)}$$

This equation is called Clasusiuss - Mossotti relation and gives α, the molecular polarizability.

Ques. - Explain Electric polarisation.

When a dielectric is placed in an external electric field, its molecule gain electric di-pole moment and the ~~diectric~~ dielectric is said to be polarised. The electric di-pole moment induced per unit volume of the dielectric material is called the electric polarisation of the dielectric. It is represented by a vector $\vec{P}$.

Let us consider a rectangular block of the polarised dielectric of length l and uniform cross-sectional area A. Let surface charge densities of fictitious charges appearing at the end faces be $+\sigma_p$ and $-\sigma_p$.



(fig.)

The charge induced on each face $= \sigma_p A$. Total induced electric di-pole moment $= \sigma_p A l$. volume of the block $= Al$. Induced di-pole moment per unit volume $= \frac{\sigma_p A l}{A l} = \sigma_p$. By definition, this therefore polarisation $P = \sigma_p$.

Thus the polarisation may also be defined as the surface density of charge appearing at faces perpendicular to the direction of applied field. Clearly the unit of polarisation is coul./m^2.

—x—

Ques:- Explain the terms electronic polarisation, atomic or ionic polarisation and dipolar polarisation.

Electronic polarisation - When a dielectric is placed in an external electric field, there is displacement of electron-cloud relative to nucleus in the atoms ~~forim~~ forming the molecules. This causes an induced di-pole moment in molecules. This phenomenon is called electronic polarisation.

Atomic or Ionic polarisation - when a dielectric, made of molecules having separate positive and negative atoms (i.e. ions), is placed in an external electric field, the separation between positive and negative charges is altered. This causes an induced di-pole moment in the molecules. This phenomenon is called Atomic or Ionic polarisation, orientational