1·7. Energy of a Plane progressive wave

When a plane harmonic waves, starting from a source of disturbance travel in a medium, the particles of the medium are successively set in vibration. The energy from the source is thus transmitted through the medium in the direction of the wave. At any instant the energy of the medium is partly kinetic and partly potential. We have to calculate the total energy per unit volume of the medium.

Let a plane longitudinal wave is travelling in a long tube of unit area of cross-section. In the first place we shall determine the kinetic energy of an oscillating layer per unit area of the plane parallel to the wave front. Let the simple harmonic wave be represented by

$$y = a \sin \frac{2\pi}{\lambda} (vt - x).$$

Differentiating it with respect to t, we obtain the velocity of all the particles in the oscillating layer.

$$\therefore \quad U = \frac{dy}{dt} = \frac{2\pi v}{\lambda} a \cos \frac{2\pi}{\lambda} (vt - x) \quad \ldots(1{\cdot}18)$$

If ρ be the undistributed density of the medium, the mass of unit area of a layer of thickness ∂x is $\rho \partial x$.

$\therefore$ K. E. of the layer $\partial k = \frac{1}{2} \times \text{mass} \times (\text{velocity})^2$

$$= \frac{1}{2} \times \rho \partial x \times \left(\frac{dy}{dt}\right)^2$$

$$= \frac{1}{2} \rho \partial x \left(\frac{2\pi a v}{\lambda}\right)^2 \cos^2 \frac{2\pi}{\alpha} (vt - x) \quad \ldots(1{\cdot}19)$$

The average kinetic energy of the whole wave of length l and unit cross-section is given by

$$K = \frac{1}{2} \rho \left(\frac{2\pi a v}{\lambda}\right)^2 \int_0^l \cos^2 \frac{2\pi}{\lambda} (vt - x) dx$$

$$= \frac{1}{2} \rho \left(\frac{2\pi a v}{\lambda}\right)^2 \times \frac{1}{2} \int_0^l \left[1 + \cos \frac{4\pi}{\lambda} (vt - x) \right] dx$$

$$= \frac{1}{4} \rho \left(\frac{2\pi a v}{\lambda}\right)^2 l \quad \ldots(1{\cdot}20)$$

$$\left[\because \int_0^l \cos \frac{4\pi}{\lambda} (vt - x) dx = 0\right]$$

Hence the average kinetic energy per unit area per unit length i.e., per unit volume of the wave is given by

$$E_k = \frac{1}{4} \rho \left(\frac{2\pi a v}{\lambda}\right)^2 \quad \ldots(1{\cdot}21)$$