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For T.D.C. Part I

Paper - 2

2 - Dimensions Geometry

SYSTEM OF CIRCLES

Q. Find the condition that the circles
 $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$
intersect orthogonally. (M. U. 1979, '81, '84, 'B. U. '85)

Solution. Let the two circles have their centres at A and B and cut at P . If the circles are orthogonal, then $AP \perp PB$.

$$\therefore AB^2 = AP^2 + PB^2 \quad \dots (1)$$

Now the centres of the two circles are $(-g_1, -f_1)$ and $(-g_2, -f_2)$ respectively.

Also their radii are

$$\sqrt{g_1^2 + f_1^2 - c_1},$$

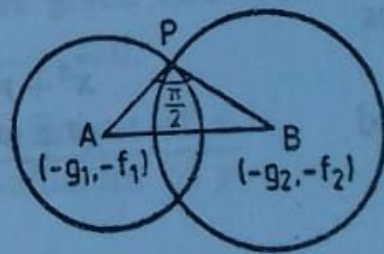
and $\sqrt{g_2^2 + f_2^2 - c_2}$ respectively.

\therefore The condition (1) becomes

$$\begin{aligned} (-g_1 + g_2)^2 + (-f_1 + f_2)^2 &= g_1^2 + f_1^2 - c_1 + g_2^2 + f_2^2 - c_2 \\ \text{or } g_1^2 + g_2^2 - 2g_1g_2 + f_1^2 + f_2^2 - 2f_1f_2 &= g_1^2 + f_1^2 - c_1 + g_2^2 + f_2^2 - c_2, \end{aligned}$$

$$\text{or } 2g_1g_2 + 2f_1f_2 = c_1 + c_2.$$

This is the required condition.



Qy prove that the radical axis of two circles is perpendicular to the line joining the centres of the circles.

[M. U. 1985, '87, '90; B.U. '85; Bh. U. '86]

Let the two circles have for their equations

$$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0 \quad \dots (1)$$

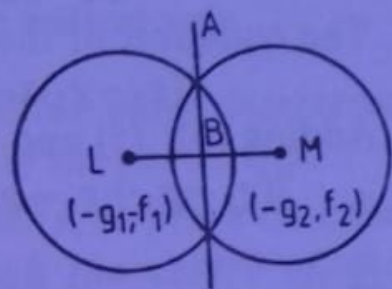
and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0. \quad \dots (2)$

The equation of their radical axis

AB is

$$2(g_1 - g_2)x + 2(f_1 - f_2)y + c_1 - c_2 = 0.$$

The slope (or gradient) of the radical



axis $AB = -\frac{\text{co-efficient of } x}{\text{co-efficient of } y}$

$$= -\frac{2(g_1 - g_2)}{2(f_1 - f_2)} = -\frac{(g_1 - g_2)}{f_1 - f_2}.$$

The co-ordinates of the centres L and M of the circles (1) and (2) are respectively $(-g_1, -f_1)$ and $(-g_2, -f_2)$.

The slope of the line LM of centres

$$= \frac{\text{Difference of } y\text{-coordinates}}{\text{Difference of } x\text{-coordinates in the same order}}$$

$$= \frac{-f_2 - (-f_1)}{-g_2 - (-g_1)} = \frac{f_1 - f_2}{g_1 - g_2}.$$

\therefore The slope of $AB \times$ the slope of LM

$$= -\frac{(g_1 - g_2)}{f_1 - f_2} \times \frac{f_1 - f_2}{g_1 - g_2} = -1.$$

Hence the radical axis of two circles is perpendicular to the line joining their centres.

or

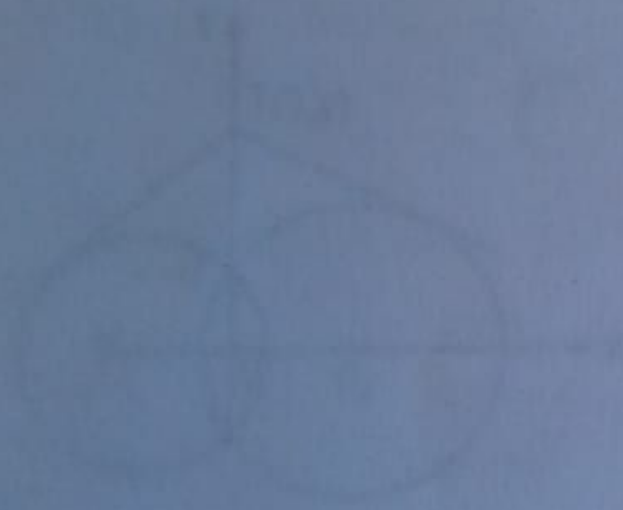
$$2x \cdot 0 + 2y \cdot 0 + 0 = 0$$

or

$$0 = 0, \text{ which is true.}$$

Hence the three radical axes of the three circles, taken in pairs, meet in a point.

The point is called the radical centre of the three circles.



Q. The radical axes of three circles, taken in pairs, are concurrent. (R. U. 1967; P. U. '68)

Solution. Let the three circles be

$$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$$

$$x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$$

and $x^2 + y^2 + 2g_3x + 2f_3y + c_3 = 0.$

Radical axes of (1), (2); (2), (3)

and (3), (1) are respectively

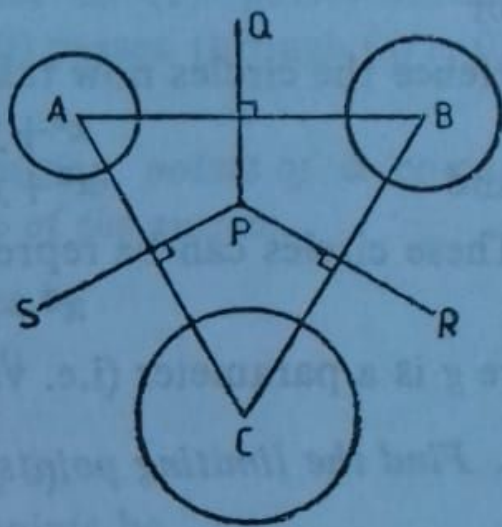
$$2x(g_1 - g_2) + 2y(f_1 - f_2) + c_1 - c_2 = 0 \quad \dots (4)$$

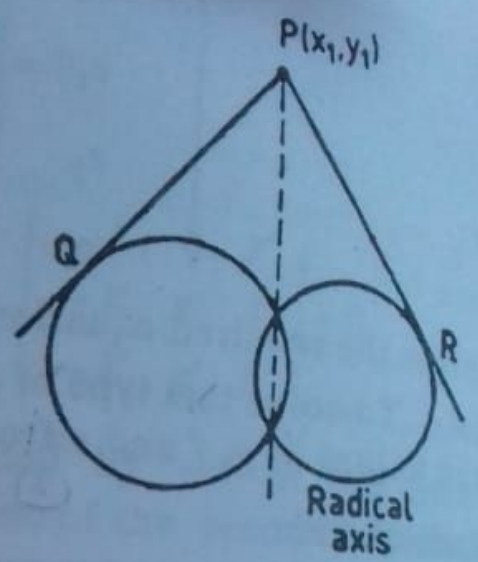
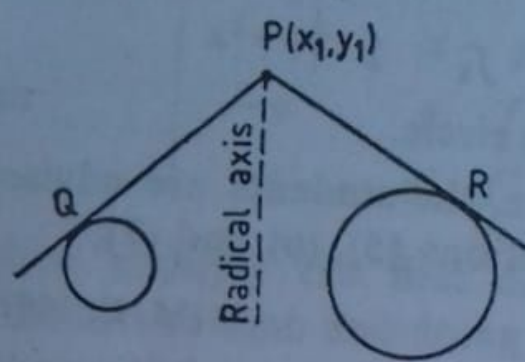
$$2x(g_2 - g_3) + 2y(f_2 - f_3) + c_2 - c_3 = 0. \quad \dots (5)$$

and $2x(g_3 - g_1) + 2y(f_3 - f_1) + c_3 - c_1 = 0. \quad \dots (6)$

Adding (4), (5) and (6) we get

$$2x(g_1 - g_2 + g_2 - g_3 + g_3 - g_1) + 2y(f_1 - f_2 + f_2 - f_3 + f_3 - f_1) + c_1 - c_2 + c_2 - c_3 + c_3 - c_1 = 0$$





Then

$$PQ = \sqrt{x_1^2 + y_1^2 + 2g_1x_1 + 2f_1y_1 + c_1}$$

and

$$PR = \sqrt{x_1^2 + y_1^2 + 2g_2x_1 + 2f_2y_1 + c_2}$$

We have

$$PQ = PR$$

or

$$PQ^2 = PR^2$$

or

$$x_1^2 + y_1^2 + 2g_1x_1 + 2f_1y_1 + c_1 = x_1^2 + y_1^2 + 2g_2x_1 + 2f_2y_1 + c_2$$

or

$$2(g_1 - g_2)x_1 + 2(f_1 - f_2)y_1 + c_1 - c_2 = 0.$$

Hence the locus of (x_1, y_1) is

$$2(g_1 - g_2)x + 2(f_1 - f_2)y + c_1 - c_2 = 0.$$

This is the required equation of the radical axis. Evidently represents a straight line, as it is of the first degree in x and y .