M.Sc. (II Sem)

Topic:→ Electromagnetic Fields Of An Accelerated Charged particle

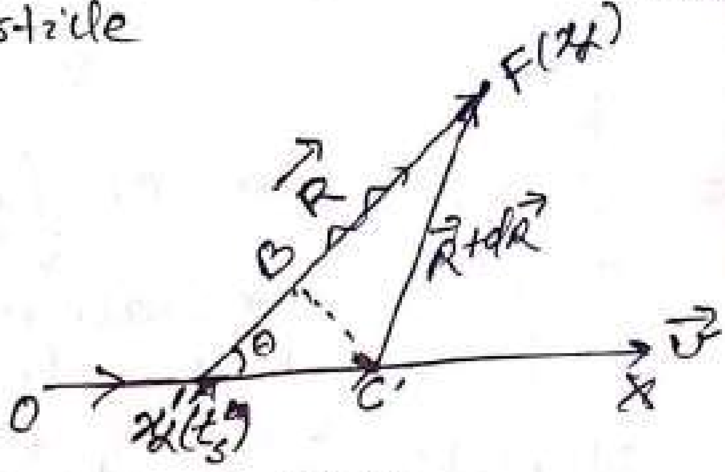

Fig (1)

Consider a charged particle is moving with velocity $\vec{v}$ on a path OX and ~~emits~~ emits radiation (electromagnetic wave) in all direction with speed C as shown in fig (1). Let time taken by emw in ~~reachi~~ reaching at field point $F(x_\mu)$ be R/C. During this period, the charged particle reaches at C, so that $AC = vR/C = \beta R$. where $\vec{\beta} = \frac{\vec{v}}{c}$

from fig (1), it is clear that

$AB = \beta R \cos\theta = \vec{\beta}\cdot\vec{R}$ and $BC = \beta R \sin\theta = \vec{\beta}\times\vec{R}$

Here, $A(x'_\mu) = A(x'_1, x'_2, x'_3)$ is the retarded position and C is the present position of charged particle corresponding to present position of electromagnetic field at $F(x_\mu) = F(x_1, x_2, x_3)$.

Therefore, $BF = AF - AB = R - \vec{R}\cdot\vec{\beta}$ and is known as retarded distance S

$$\therefore S = BF = R - \vec{R}\cdot\vec{\beta} \longrightarrow (1)$$

In terms of retarded distance S, the Lienard-Wiechert potential are given as

$$\phi = \frac{e}{4\pi\epsilon_0}\frac{1}{[R-\vec{\beta}\cdot\vec{R}]} = \frac{e}{4\pi\epsilon_0}\cdot\frac{1}{S} \quad \cdots (a)$$

and

$$\vec{A} = \frac{\mu_0 e}{4\pi}\left[\frac{\vec{v}}{R-\vec{\beta}\cdot\vec{R}}\right] = \frac{e}{4\pi\epsilon_0}\cdot\frac{\vec{v}}{c^2 S} \quad \cdots (b)$$

(2)

As, in case of an accelerated charge, it is not possible to express the potential in terms of the present position only, the components of ∇ are partial derivatives at constant time t but not at constant time t'. Since the time variation with respect to t' is given, In order to compute the fields, we have to transform $\frac{\partial}{\partial t}\Big|_{x_\alpha}$ and $\nabla\Big|_{x_\alpha}$ to expression in terms of $\frac{\partial}{\partial t'}\Big|_{x_\alpha}$.

For this, we have from fig (1)

$$R(x_\alpha, x'_\alpha) = \sqrt{(x_1-x'_1)^2+(x_2-x'_2)^2+(x_3-x'_3)^2}$$

$$= \left[\sum(x_\alpha - x'_\alpha)^2\right]^{1/2} = c(t-t_s) \longrightarrow (3)$$

(Here subscript s denotes the retardation)

$\because x'_\alpha$ is given as a function of retarded time t_s, R is function of x_α and t_s

i.e. $R(x_\alpha, x'_\alpha) = f(x_\alpha, t_s) = c(t-t_s) \longrightarrow (4)$

Thus we have two relation as

$\rightarrow$ when $R = c(t-t_s)$

$$\therefore \frac{\partial R}{\partial t} = c\left(1-\frac{\partial t_s}{\partial t}\right) \longrightarrow (5)$$

Also, $$\frac{\partial R}{\partial t} = \frac{\partial R}{\partial t_s}\cdot\frac{\partial t_s}{\partial t} \longrightarrow (6)$$

from fig (1) we have

$$\vec{v}\,\partial t_s + \vec{R} + \partial\vec{R} = \vec{R} \Rightarrow, \quad \frac{\partial \vec{R}}{\partial t_s} = -\vec{v} \longrightarrow (7)$$

Again $R^2 = \vec{R}\cdot\vec{R}$ $\therefore 2R\frac{\partial R}{\partial t_s} = 2\vec{R}\cdot\frac{\partial \vec{R}}{\partial t_s}$

$$\therefore \frac{\partial R}{\partial t_s} = \frac{\vec{R}}{R}\cdot(-\vec{v})$$

$$= -\frac{\vec{R}\cdot\vec{v}}{R} \longrightarrow (8)$$

from eq^n (6) and (8), we have

$$\frac{\partial R}{\partial t} = -\frac{\vec{R}\cdot\vec{v}}{R}\cdot\frac{\partial t_s}{\partial t} \longrightarrow (9)$$

from eq^n (5) and (9), we have

$$c\left(1-\frac{dt_s}{dt}\right) = -\frac{\vec{R}\cdot\vec{v}}{R}\cdot\frac{\partial t_s}{\partial t}$$

or, $$c - c\frac{\partial t_s}{\partial t} = -\frac{\vec{R}\cdot\vec{v}}{R}\cdot\frac{\partial t_s}{\partial t}$$

or, $$c = \left(c - \frac{\vec{R}\cdot\vec{v}}{R}\right)\frac{\partial t_s}{\partial t}$$

or, $$1 = \left(1 - \frac{\vec{R}}{R}\cdot\vec{\beta}\right)\frac{\partial t_s}{\partial t}$$

or, $$\frac{\partial t_s}{\partial t} = \frac{R}{(R-\vec{R}\cdot\vec{\beta})} = \left(\frac{R}{R-\vec{R}\cdot\vec{\beta}}\right)\frac{\partial t_s}{\partial t_s} \quad \left(\because \frac{\partial t_s}{\partial t_s} = 1\right)$$

$$= \frac{R}{S}\cdot\frac{\partial t_s}{\partial t_s}$$

$$\therefore \boxed{\frac{\partial}{\partial t} \rightarrow \frac{R}{S}\frac{\partial}{\partial t_s}} \longrightarrow (10)$$

→ when $R = R(x_\alpha, t_s)$

$$\therefore \nabla R = \frac{\partial R}{\partial x_\alpha}\Big|_{t_s} dx_\alpha + \left(\frac{\partial R}{\partial t_s}\right)_{x_\alpha} dt_s \longrightarrow (10a)$$

$$= \nabla_s R + \left(-\frac{\vec{R}\cdot\vec{v}}{R}\right)\nabla t_s \quad \text{from (8)}$$

where, $\nabla_s = \frac{\partial}{\partial x_\alpha} dx_\alpha$

$\longrightarrow (11)$

$R^2 = \sum(x_\alpha - x'_\alpha)^2$

$\therefore 2R\frac{\partial R}{\partial x_\alpha} = \sum 2(x_\alpha - x'_\alpha)\cdot 1 \quad \therefore \frac{\partial R}{\partial x_\alpha} = \frac{\sum(x - x'_\alpha)}{R} = \frac{\vec{R}}{R} = \hat{R}$

$$\therefore \nabla R = \nabla_s R - \frac{\vec{R}\cdot\vec{v}}{R}\nabla t_s = \hat{R} - (\hat{R}\cdot\vec{v})\nabla t_s \longrightarrow (12)$$

Again as $R = c(t - t_s) \quad \therefore \nabla R = \nabla c(t - t_s)$

$\therefore \nabla R = -c\nabla t_s \quad \because \nabla t = 0$

Equating these two equation we have.

$$-c\nabla t_s = \hat{R} - (\hat{R}\cdot\vec{v})\nabla t_s$$

$$\text{or} \quad (\hat{R}\cdot\vec{v} - c)\nabla t_s = \hat{R}$$

$$\therefore \nabla t_s = \frac{\hat{R}}{-c + \hat{R}\cdot\vec{v}} = -\frac{\hat{R}}{-c + \hat{R}\cdot\vec{v}}$$

$$= -\frac{1}{c}\left(\frac{\hat{R}}{1 - \hat{R}\cdot\vec{\beta}}\right) \longrightarrow (13) = -\frac{\vec{R}}{cs}$$

putting the value of ∇t_s in eq^n-(12), we have

$$\nabla R = \nabla_s R + \left(\frac{\partial R}{\partial t_s}\right)\cdot\nabla t_s$$

$$= \nabla_s R - \frac{1}{c}\left(\frac{\hat{R}}{1 - \hat{R}\cdot\vec{\beta}}\right)\left(\frac{\partial R}{\partial t_s}\right)$$

$$= \nabla_s R - \frac{1}{c}\left(\frac{\vec{R}}{R - \vec{\beta}\cdot\vec{R}}\right)\left(\frac{\partial R}{\partial t_s}\right)$$

$$= \nabla_s R - \frac{\vec{R}}{cs}\frac{\partial R}{\partial t_s}$$

$$\therefore \nabla = \nabla_s - \frac{\vec{R}}{cs}\frac{\partial}{\partial t_s} \longrightarrow (14)$$

Now the fields can be found from the relations

$$\vec{E} = -\nabla\Phi - \frac{\partial\vec{A}}{\partial t} \text{ and } \vec{B} = \vec{\nabla}\times\vec{A} \longrightarrow (15)$$

(5)

where ~~poten~~ scalar and vector magnetic potentials

$$\Phi = \frac{e}{4\pi\epsilon_0}\frac{1}{[R-\vec{\beta}\cdot\vec{R}]} = \frac{e}{4\pi\epsilon_0}\cdot\frac{1}{S}$$

and $$\vec{A} = \frac{\mu_0}{4\pi} e\left[\frac{v}{R-\beta\cdot\vec{R}}\right] = \frac{e}{4\pi\epsilon_0}\frac{v}{c^2 S}$$

Thus first of equation (15) becomes

$$\vec{E} = -\frac{e}{4\pi\epsilon_0}\cdot\nabla\left(\frac{1}{S}\right) - \frac{e}{4\pi\epsilon_0}\frac{\partial}{\partial t}\left(\frac{v}{Sc^2}\right)$$

$$= \frac{e}{4\pi\epsilon_0}\left[\frac{1}{S^2}\nabla_s S - \frac{\vec{R}}{S^3 c}\frac{\partial S}{\partial t_s} - \frac{R}{S^2c^2}\vec{v} + \frac{Rv}{c^2S^3}\frac{\partial S}{\partial t_s}\right] \longrightarrow (16)$$

$$= \frac{e}{4\pi\epsilon_0}\left[\frac{\vec{R}}{S^2 R} - \frac{\vec{v}}{cS^2} + \frac{R}{S^3c}\left(\frac{\vec{R}\cdot\vec{u}}{R}\right) - \frac{R}{S^3c}\frac{u^2}{c} + \frac{R}{S^3c}\left(\frac{\vec{R}\cdot\dot{\vec{v}}}{c}\right)\right.$$

$$\left. - \frac{R\dot{v}}{S^2c^2} - \frac{R}{S^3c^2}v\left(\frac{\vec{R}\cdot\vec{u}}{R}\right) + \frac{R}{S^3c^3}\vec{v}v^2 - \frac{R}{c^2S^3}\vec{v}\left(\frac{\vec{R}\cdot\dot{\vec{v}}}{c}\right)\right] \longrightarrow (17)$$

$$\left(\because \nabla_s S = \frac{\vec{R}}{R} - \frac{v}{c}\right)$$

Rearranging and Combining the terms, we have

$$\vec{E} = \frac{e}{4\pi\epsilon_0}\left[\frac{1}{S^3}\left(\vec{R} - \frac{R\vec{v}}{c}\right)\left(1-\frac{v^2}{c^2}\right) + \frac{1}{c^2S^3}\left\{\vec{R}\times\left(\left(\vec{R}-\frac{R\vec{v}}{c}\right)\times\dot{\vec{v}}\right)\right\}\right] \longrightarrow (18)$$

Similarly

$$\vec{B} = \frac{e}{4\pi\epsilon_0 c^2}\left[\frac{\vec{v}\times\vec{R}}{S^3}\left(1-\frac{v^2}{c^2}\right) + \frac{1}{cS^3}\frac{\vec{R}}{R}\times\left\{\vec{R}\times\left(\left(\vec{R}-\frac{R\vec{v}}{c}\right)\times\dot{\vec{v}}\right)\right\}\right] \longrightarrow (19)$$

These eq^n (18) and (19) give us

$$\vec{B} = \frac{\vec{R}\times\vec{E}}{Rc} \longrightarrow (20)$$

Thus the magnetic field $\vec{B}$ is always $\perp$ to

$\vec{R}$ and $\vec{E}$. Examining above two equations (6) we find that the first component given by the first term is a function of velocity $\vec{v}$ while the second term is a function of acceleration $\dot{\vec{v}}$.

We can, therefore write

$$\vec{E} = \vec{E}_v + \vec{E}_a \longrightarrow (21)$$

where $\vec{E}_v$ is the velocity field and $\vec{E}_a$ the acceleration field. We further see that $\vec{E}_v \propto \frac{1}{R^2}$ while $\vec{E} \propto \frac{1}{R}$. Therefore poynting vector for the fields, are

$$\vec{N}_v \propto \frac{1}{R^4} \quad \text{and} \quad \vec{N}_a \propto \frac{1}{R^2}$$

Thus, ~~fine~~ for larg R, the Contribution due to $\vec{N}_v$ tends to zero while due to $\vec{N}_a$ is finite. Hence we conclude that a particle moving with a uniform velocity cannot ~~be~~ radiate energy, but ~~Energ~~ energy can be radiated only by accelerated charges. The radiation produced by deceleration is called "Bremsstrahlung".

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