The average value of magnetic moment in field direction is

$$< \mu \cos \theta > = \int_0^{\pi} \mu \cos\theta \, dn \; / \int_0^{\pi} dn \qquad (3)$$

$$< \mu \cos \theta > = \int_0^{\pi} \mu \cos\theta \, 2\pi \sin\theta \, d\theta \, e^{\mu H \cos\theta / KT} \; / \int_0^{\pi} 2\pi \sin\theta d\theta \, e^{\mu H \cos\theta / KT}$$

$$= \mu \int_0^{\pi} \cos\theta \; \sin\theta \, d\theta \, e^{\mu H \cos\theta / KT} \; / \int_0^{\pi} \sin\theta d\theta \, e^{\mu H \cos\theta / KT} \qquad (4)$$

In order to evaluate the integral , let

$X = \mu H \cos\theta / KT$ and $\mu H / KT = a$ (5)

$\cos\theta = KTx / \mu H$

$\therefore \quad -\sin\theta d\theta = KTdx / \mu H$

When $\theta = 0$, $x = \mu H / KT = +a$

When $\theta = 0$, $x = -\mu H / KT = -a$

Putting these values in (4), we get

$$< \mu \cos\theta > \;=\; \mu \int_{+a}^{-a} \frac{KT}{\mu H}\, x \left( -\frac{KTdx}{\mu H} \right) e^{x} \Big/ \int_{+a}^{-a} \left( -\frac{KTdx}{\mu H} \right) e^{x} \qquad (6)$$

$$= \mu \text{ X } 1/a \int_{-a}^{+a} x\, e^{x}\, dx \Big/ \int_{-a}^{+a} e^{x}\, dx$$

$$= \mu \text{X} 1/a\, [\, xe^{x} - e^{x}\, ]_{-a}^{+a} \Big/ [\, e^{x}\, ]_{-a}^{+a}$$

$$< \mu \cos\theta > = \mu/a \, [ \, (a e^{a} + a e^{-a}) - ( e^{a} - e^{-a} ) \, ] / ( e^{a} - e^{-a} )$$

$$< \mu \cos\theta > = \mu/a \, [ \, a \, (e^{a} + e^{-a}) - ( e^{a} - e^{-a} ) \, ] / ( e^{a} - e^{-a} )$$

$$< \mu \cos\theta > = \mu/a \, [ \, a \, (e^{a} + e^{-a}) / ( e^{a} - e^{-a} ) - 1 \, ]$$

$$< \mu \cos\theta > = \mu \, [ \, a \, (e^{a} + e^{-a}) / ( e^{a} - e^{-a} ) - 1/a \, ]$$

Hence $< \mu \cos\theta > = \mu [ \cot ha - 1/a ]$

Where $\cot ha = (e^{a} + e^{-a})/(e^{a} - e^{-a})$

$\therefore \quad < \mu \cos\theta > = \mu L(a) \qquad (7)$

Where $L(a) = [ \cot ha - 1/a ]$ = Langevin's function

Now plot a graph between Langevin function L (a) and ($a = \mu H / KT$) as shown in fig below . here we obtain a curve . This curve is known as Langevin curve .