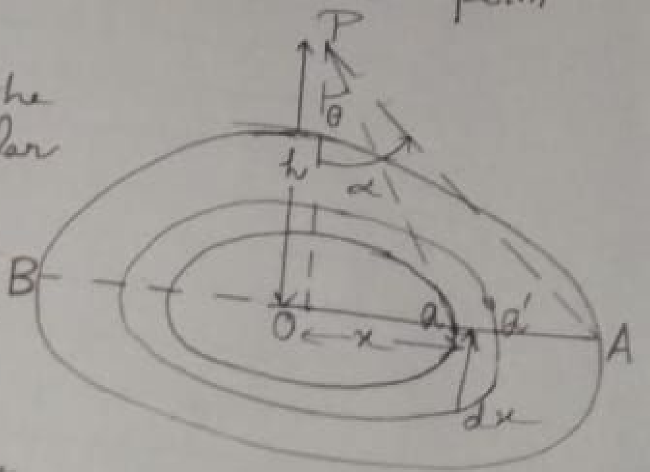
Attraction and Potential

Q) Find the attraction of a thin uniform circular disc at a point on its axis.



Let O be the centre of a circular plate of uniform thickness k and density ρ. Let its radius be a.

Let P be any point on its axis and suppose $OP = h$

We first find the attraction at P of a thin circular Concentric ring of radius x and breadth dx.

Let Q be any point on this ring. The attraction at P of a unit mass at Q is $\frac{\gamma}{PQ^2}$ which is the same for all points Q on the ring. The direction of this attraction is along PQ. Its resolved parts along and perpendicular to PO in the plane POQ are respectively $\frac{\gamma}{PQ^2}\cos\theta$ and $\frac{\gamma}{PQ^2}\sin\theta$

where $\theta = \angle OPQ$ which is the same as for all points Q on the ring.

If we sum for attraction at P due to the whole ring, we find that the sum of the resolved parts perpendicular to PO is zero while the sum of the resolved parts along PO.

$$= \left(\frac{\gamma}{PQ^2}\cdot\cos\theta\right)\cdot(\text{mass of the ring})$$

$$= \gamma\cdot\frac{PO}{PQ^3}\cdot 2\pi x\,dx\cdot k\rho$$

$$= \frac{2\pi\gamma k\rho\cdot h\tan\theta\cdot h\sec^2\theta\,d\theta}{(h\sec\theta)^3}$$

$$= 2\pi\gamma k\rho\sin\theta\,d\theta$$

Since the given disc can be supposed to be composed of concentric rings.

$\therefore$ The attraction at P due to the whole disc is along PO and its magnitude.

$$= 2\pi\gamma k\rho\int_0^{\alpha}\sin\theta\,d\theta$$

(where α is the angle subtended at P by a radius of the disc.)

$$= 2\pi \gamma k \rho \,(1 - \cos\alpha)$$

$$= \frac{2M\gamma}{a^2}\,(1 - \cos\alpha)$$

where M is the mass of the disc.

<u>Note</u> :- 1 If the disc is infinite then $\alpha \to \frac{\pi}{2}$ and hence the attraction at P of an infinite disc is along PO and its magnitude is $2\pi\gamma k\rho$ which is independent of the position of P.

2 If the radius of the disc is finite and P is very near O, then $\alpha \to \frac{\pi}{2}$ and hence the attraction at P is again $2\pi\gamma k\rho$.