Derive Wien's displacement ~~formuud~~ Law for distribution of energy in the spectrum emitted by a blackbody.

<u>Wien's Displacement Law</u>:. The Planck's radiation formula is

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda KT}-1}$$

or, $$u_\lambda = 8\pi hc\,(\lambda^{-5})\left(e^{hc/\lambda KT}-1\right)^{-1}.$$

To find the wavelength at which the spectral radiancy is maxm, we put

$$\frac{du_\lambda}{d\lambda} = 0$$

that is

$$8\pi hc\left[-5(\lambda^{-6})\left(e^{hc/\lambda KT}-1\right)^{-1} + \lambda^{-5}(-1)\left(e^{hc/\lambda KT}-1\right)^{-2} e^{hc/\lambda KT}\left(\frac{-hc}{\lambda^2 KT}\right)\right] = 0$$

or, $$\frac{5}{\lambda} = \left(e^{hc/\lambda KT}-1\right)^{-1} e^{hc/\lambda KT}\,\frac{hc}{\lambda^2 KT}$$

or, $$5 = \frac{hc}{\lambda KT}\,\frac{e^{hc/\lambda KT}}{e^{hc/\lambda KT}-1}$$

Putting $\frac{hc}{\lambda KT} = x$, We get

$$5 = \frac{x e^x}{e^x - 1}$$

$$= \frac{x}{1 - e^{-x}}$$

or, $$\frac{x}{5} + e^{-x} = 1$$

This equation has a single root given by $x = 4.965$, and therefore x must be a constant. That is

$$\frac{hc}{\lambda KT} = 4.965$$

Therefore, the wavelength λ_m at which the spectral radiancy per unit range of wavelength has its max^{m} value is given by

$$\lambda_m T = \frac{hc}{4.965\ K} = b \text{ (say)}.$$