VECTOR - ALGEBRA

Scalar triple product of vectors and its geometrical meaning.

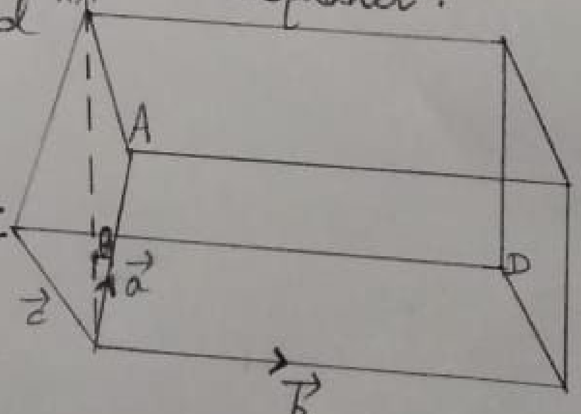
DEFINITION :-

Let $\vec{a}, \vec{b}, \vec{c}$ be vectors. then the scalar product of any one of them with the vector product of the other two is called a triple product of the vectors $\vec{a}, \vec{b}, \vec{c}$.

Thus $\vec{a}\cdot\vec{b}\times\vec{c}$, $\vec{b}\cdot\vec{c}\times\vec{a}$, $\vec{c}\cdot\vec{a}\times\vec{b}$, $\vec{c}\cdot\vec{b}\times\vec{a}$ etc. are scalar triple product of $\vec{a}, \vec{b}, \vec{c}$.

Geometrical Meaning of $\vec{a}\cdot\vec{b}\times\vec{c}$:-

For the sake of generality suppose $\vec{a}, \vec{b}, \vec{c}$ are non-coplaner. Construct a parallelopiped with $\vec{a}, \vec{b}, \vec{c}$ as coterminus edges.



Then with reference to the self-explanatory adjoing figure we have

$\vec{b}\times\vec{a}$ = area of the parallelogram OBDC) $\vec{n}$

where $\vec{n}$ = a unit vector perpendicular to the plane OBDC)

Let $\vec{n}$ make an angle θ with $\vec{a}$.

Then $\vec{a}\cdot(\vec{b}\times\vec{c}) = \vec{b}\times\vec{c}\cdot\vec{a}$

$= (\text{area } OBDC)\,\vec{n}\cdot\vec{a}$

$= (\text{area } OBDC)$ (projection of $\vec{OA}$ on $\vec{n}$).

$= (\text{area } OBDC)\cdot OH$

= area of the base OBDC of the parallelopiped ODA).(height OH of the parallelopiped ODA)

= Volume of the parallelopiped ODA.

= Volume of the parallelopiped with $\vec{a}, \vec{b}, \vec{c}$ as coterminus edges.

Thus geometrically, $\vec{a}\cdot\vec{b}\times\vec{c}$ denotes the volume of the parallelopiped formed with $\vec{a}, \vec{b}, \vec{c}$ as co-terminus edges.