Problems :-

Q ① If $f(x, y, z) = 3x^2y - y^3z^2$ find grad f at the point $(1, -2, 1)$

Sol:- By definition

$$\text{grad } f = \nabla f$$

But $$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

$$\therefore \text{grad } f = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) f$$

$$= \vec{i}\frac{\partial f}{\partial x} + \vec{j}\frac{\partial f}{\partial y} + \vec{k}\frac{\partial f}{\partial z}$$

$\because f = 3x^2y - y^3z^2$

$$\therefore \frac{\partial f}{\partial x} = 6xy.$$

$$\frac{\partial f}{\partial y} = 3x^2 - 3y^2z^2$$

$$\frac{\partial f}{\partial z} = 2y^3z$$

$$\therefore \text{grad } f = 6xy\,\vec{i} + 3(x^2 - y^2z^2)\vec{j} + 2y^3z\,\vec{k}$$

Hence at $(1, -2, 1)$

$$\text{grad } f = -12\vec{i} - 9\vec{j} - 16\vec{k}$$

② If $\phi(x,y,z) = x^2y + y^2x + z^2$ find $\nabla\phi$ at the point $(1,1,1)$.

Sol:

By definition

$$\text{grad}\,\phi = \nabla\phi$$

But $$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

$$\therefore \text{grad}\,\phi = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)\phi$$

$$\text{grad}\,\phi = \vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}$$

$$\phi = x^2y + y^2x + z^2$$

$$\frac{\partial\phi}{\partial x} = 2xy + y^2$$

$$\frac{\partial\phi}{\partial x} = x^2 + 2y$$

$$\frac{\partial\phi}{\partial z} = 2z$$

$$\therefore \text{grad}\,\phi = (2xy + y^2)\vec{i} + (x^2 + 2y)\vec{j} + 2z\vec{k}$$

Hence at $(1,1,1)$

$$\nabla\phi = 3\vec{i} + 3\vec{j} + 2\vec{k}$$