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VECTOR CALCULUS

Q) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then $(\vec{a}\cdot\nabla)\vec{r} = \vec{a}$

Let $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

Then $\vec{a}\cdot\nabla = (a_1\vec{i} + a_2\vec{j} + a_3\vec{k})\cdot\left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)$

$$= a_1\frac{\partial}{\partial x} + a_2\frac{\partial}{\partial y} + a_3\frac{\partial}{\partial z}$$

$$\therefore (\vec{a}\cdot\nabla)\vec{r} = \left(a_1\frac{\partial}{\partial x} + a_2\frac{\partial}{\partial y} + a_3\frac{\partial}{\partial z}\right)(x\vec{i} + y\vec{j} + z\vec{k})$$

$$= a_1\vec{i} + 0 + 0 + 0 + a_2\vec{j} + 0 + 0 + 0 + a_3\vec{k}$$

$$= \vec{a}$$

Q.) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$, then

$$\nabla r^n = nr^{n-2}\vec{r}$$

we have

$$\nabla r^n = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)r^n$$

$$= \vec{i}\left(nr^{n-1}\frac{\partial r}{\partial x}\right) + \vec{j}\left(nr^{n-1}\frac{\partial r}{\partial y}\right) + \vec{k}\left(nr^{n-1}\frac{\partial r}{\partial z}\right) \quad \text{—— (A)}$$

But since $|\vec{r}| = r$

$$\therefore x^2 + y^2 + z^2 = r^2$$

Differentiating partially w.r.t. x we get

$$2x = 2r\frac{\partial r}{\partial x}$$

or, $\frac{\partial r}{\partial x} = \frac{x}{r}$

similarly $\dfrac{\partial r}{\partial y} = \dfrac{y}{x}$

and $\dfrac{\partial r}{\partial z} = \dfrac{z}{r}$

$\therefore$ A gives

$$\nabla r^n = n r^{n-1}\left(\vec{i}\,\frac{x}{r} + \vec{j}\,\frac{y}{x} + \vec{k}\,\frac{z}{r}\right)$$

$$= n r^{n-2}\left(x\vec{i} + y\vec{j} + z\vec{k}\right)$$

$$= n r^{n-2}\,\vec{r}$$