VECTOR ALGEBRA

Q.) **Physical meaning of curl :-**

If $\vec{v}$ be the velocity of a particle of a rigid body whose angular velocity is $\vec{\omega}$, then curl $\vec{v} = 2\vec{\omega}$.

Proof :-

Let $\vec{r}$ be the position vector of the particle whose velocity is $\vec{v}$. Then

$\vec{v} = \vec{\omega} \times \vec{r}$ (when $\vec{\omega}$ is the given constant angular velocity of the body.)

$$\therefore \quad \text{curl } \vec{v} = \text{curl } (\vec{\omega} \times \vec{r})$$

$$= \vec{\omega} \text{ div } \vec{r} - \vec{r} \text{ div } \vec{\omega} + (\vec{r} \cdot \nabla)\vec{\omega} - (\vec{\omega} \cdot \nabla)\vec{r}$$

But div $\vec{r} = \sum \vec{i} \cdot \frac{\partial}{\partial x}(\vec{i}x + \vec{j}x + \vec{k}z)$

$$= \sum \frac{\partial x}{\partial x}$$

$$= \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z}$$

$$= 3$$

$$\left.\begin{array}{l} \text{div } \omega = 0 \\ (\vec{r} \cdot \nabla)\omega = 0 \end{array}\right\} (\because \vec{\omega} \text{ is constant vector})$$

and $(\vec{\omega} \cdot \nabla)\vec{r} = \vec{\omega}$ (by previous question)

$\therefore$ (A) gives

$$\text{curl } \vec{v} = 3\vec{\omega} - 0 + 0 - \vec{\omega}$$
$$= 2\vec{\omega}$$