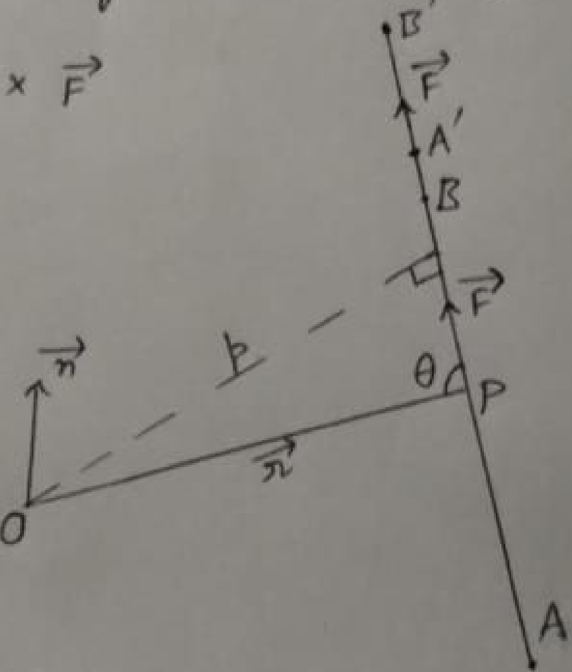
(3) Prove that the moment of a vector about a point is unaltered by sliding the vector along its line of action.

Proof :- Let O be a given point and $\vec{F} = \vec{AB}$ be any vector. Then by definition, the moment of $\vec{F}$ about O is given by

$$\vec{G} = \vec{r} \times \vec{F}$$



where $\vec{r}$ is the position vector of any point P on the line of action AB of $\vec{F}$.

$$\therefore \vec{G} = [|\vec{r}|\,|\vec{F}|\sin(\pi-\theta)]\vec{n}$$

$$= (OP \sin\theta\, F)\,\vec{n}$$

$$= pF\,\hat{n} \quad \text{———} \quad (1)$$

where

θ = angle $(\vec{r}, \vec{F})$

$\vec{n}$ = unit vector normal to the plane of $\vec{r}$ and $\vec{F}$ in the sense of a right hand rotation from $\vec{r}$ to $\vec{F}$.

p = length of perpendicular from O on the line of action of $\vec{F}$.

$F = |\vec{F}|$

Now, suppose the vector $\vec{F}$ slides along its line of action and at any instant occupies the position $A'B'$ on AB produced either way.

Then the length of perpendicular from O on $A'B'$ remains unchanged, i.e. p does not change.

Moreover $\vec{n}$ and $\vec{F}$ also do not change.

$\therefore$ (1) shows that $\vec{G}$ remains unchanged

Hence the theorem.