

Vector algebra — Problems

(3)

$$(v) (\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) + (\vec{a} \times \vec{c}) \times (\vec{d} \times \vec{b}) + (\vec{a} \times \vec{d}) \times (\vec{b} \times \vec{c}) = 2[\vec{b} \ \vec{d} \ \vec{c}] \vec{a}$$

$$\begin{aligned} \text{L.H.S.} &= [\vec{a} \ \vec{c} \ \vec{d}] \vec{b} - [\vec{b} \ \vec{c} \ \vec{d}] \vec{a} \\ &+ [\vec{a} \ \vec{d} \ \vec{b}] \vec{c} - [\vec{c} \ \vec{d} \ \vec{b}] \vec{a} \\ &+ [\vec{a} \ \vec{d} \ \vec{c}] \vec{b} - [\vec{a} \ \vec{d} \ \vec{b}] \vec{c} \\ &= [\vec{a} \ \vec{c} \ \vec{d}] \vec{b} - [\vec{b} \ \vec{c} \ \vec{d}] \vec{a} \\ &\quad - [\vec{c} \ \vec{d} \ \vec{b}] \vec{a} - [\vec{a} \ \vec{c} \ \vec{d}] \vec{b} \\ &= -[\vec{b} \ \vec{c} \ \vec{d}] \vec{a} - [\vec{b} \ \vec{c} \ \vec{d}] \vec{a} \\ &= 2[\vec{b} \ \vec{d} \ \vec{c}] \vec{a} \\ &= \text{R.H.S.} \end{aligned}$$

(vi)

$$\begin{vmatrix} \vec{a} & a_1 & a_2 & a_3 \\ \vec{b} & b_1 & b_2 & b_3 \\ -\vec{c} & c_1 & c_2 & c_3 \\ -\vec{d} & d_1 & d_2 & d_3 \end{vmatrix} = 2(\vec{c} \times \vec{d}) \times (\vec{a} \times \vec{b})$$

Where $\vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$ etc.

$$\begin{aligned} \text{L.H.S.} &= \vec{a} [\vec{b} \ \vec{c} \ \vec{d}] - \vec{b} [\vec{a} \ \vec{c} \ \vec{d}] - \vec{c} [\vec{a} \ \vec{b} \ \vec{d}] \\ &\quad + \vec{d} [\vec{a} \ \vec{b} \ \vec{c}] \\ &= -\{[\vec{a} \ \vec{c} \ \vec{d}] \vec{b} - [\vec{b} \ \vec{c} \ \vec{d}] \vec{a}\} \\ &\quad -\{[\vec{a} \ \vec{b} \ \vec{d}] \vec{c} - [\vec{a} \ \vec{b} \ \vec{c}] \vec{d}\} \\ &= -(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) - (\vec{a} \times \vec{b})(\vec{c} \times \vec{d}) \\ &= +2(\vec{c} \times \vec{d}) \times (\vec{a} \times \vec{b}) \\ &= \text{R.H.S.} \end{aligned}$$

(2)

(vii) $[\vec{b} \times \vec{c} \quad \vec{c} \times \vec{a} \quad \vec{a} \times \vec{b}] = [\vec{a} \ \vec{b} \ \vec{c}]^2$

L.H.S. $= (\vec{b} \times \vec{c}) \cdot (\vec{c} \times \vec{a}) \times \vec{a} \times \vec{b}$

$$= (\vec{b} \times \vec{c}) \cdot \{ [\vec{c} \ \vec{a} \ \vec{b}] \vec{a} - [\vec{c} \ \vec{a} \ \vec{a}] \vec{b} \}$$

$$= [\vec{c} \ \vec{a} \ \vec{b}] \vec{b} \times \vec{c} \cdot \vec{a} - 0$$

$$= [\vec{a} \ \vec{b} \ \vec{c}] [\vec{a} \ \vec{b} \ \vec{c}]$$

$$= [\vec{a} \ \vec{b} \ \vec{c}]^2$$

$=$ R.H.S.

(viii) $[\vec{a} \times \vec{b} \quad \vec{c} \times \vec{d} \quad \vec{e} \times \vec{f}] = [\vec{c} \ \vec{e} \ \vec{f}] [\vec{a} \ \vec{b} \ \vec{d}] - [\vec{d} \ \vec{e} \ \vec{f}] [\vec{a} \ \vec{b} \ \vec{c}]$

L.H.S. $= (\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) \times (\vec{e} \times \vec{f})$

$$= (\vec{a} \times \vec{b}) \cdot \{ [\vec{c} \ \vec{e} \ \vec{f}] \vec{d} - [\vec{d} \ \vec{e} \ \vec{f}] \vec{c} \}$$

$$= [\vec{c} \ \vec{e} \ \vec{f}] \vec{a} \times \vec{b} \cdot \vec{d} - [\vec{d} \ \vec{e} \ \vec{f}] \vec{a} \times \vec{b} \cdot \vec{c}$$

$$= [\vec{c} \ \vec{e} \ \vec{f}] [\vec{a} \ \vec{b} \ \vec{d}] - [\vec{d} \ \vec{e} \ \vec{f}] [\vec{a} \ \vec{b} \ \vec{c}]$$

(ix) $(\vec{a} \times \vec{b}) \times (\vec{a} \times \vec{c}) \cdot \vec{d} = \vec{a} \cdot \vec{d} [\vec{a} \ \vec{b} \ \vec{c}]$

L.H.S. $= \{ [\vec{a} \ \vec{b} \ \vec{c}] \vec{a} - [\vec{a} \ \vec{b} \ \vec{a}] \vec{c} \} \cdot \vec{d}$

$$= [\vec{a} \ \vec{b} \ \vec{c}] \vec{a} \cdot \vec{d} - 0$$

$=$ R.H.S.