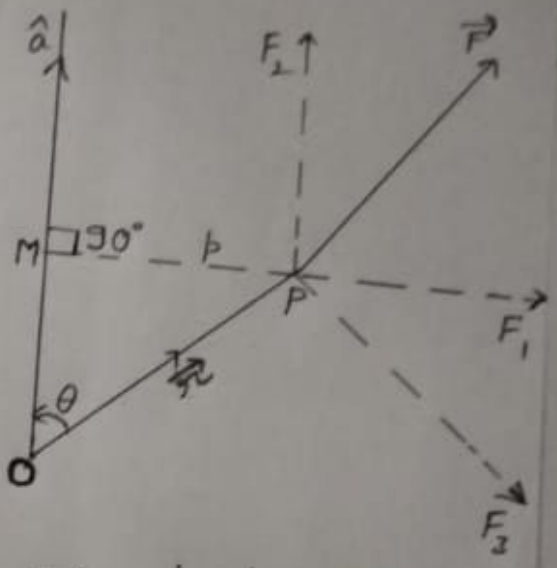
Q. 2) Define the scalar moment of a (localised) vector about an axis.

Let F be a localised vector acting at any point P. Let OM be any given axis specified by the unit vector $\hat{a}$.



Let $p = PM$ be perpendicular from P to the axis.

With respect to any point O in the given axis let $\vec{r}$ be the position vector of P. Let $|\vec{r}| = r$ and $\angle POM = \theta$.

Let the Components of $\vec{F}$ be $[F_1, F_2, F_3]$ in directions along $\overrightarrow{MP}$, parallel to $\hat{a}$, and perpendicular to both $\overrightarrow{MP}$ and $\hat{a}$ respectively.

Now the line of action of F_1 intersects the axis, and the line of action of F_2 is parallel to the axis.

Therefore F_1 and F_2 have no turning effect about $\hat{a}$, the Component F_3 has a turning effect about $\hat{a}$ and this turning effect is clearly pF_3.

since $\hat{a} \times \vec{r}$ is a vector having magnitude $r\sin\theta$, i.e. p, and its direction is parallel to that of the component F_3, we have

$$pF_3 = (r\sin\theta)\, F_3$$
$$= (\hat{a} \times \vec{r}) \cdot \vec{F}$$
$$= [\hat{a}\ \vec{r}\ \vec{F}]$$

Thus the turning effect of $\vec{F}$ about the axis $\hat{a}$ is given by the scalar triple product $[\hat{a}\ \vec{r}\ \vec{F}]$ where $\vec{r}$ is the position vector of $\vec{F}$ w.r.t. any point O on the axis $\hat{a}$.

The scalar triple product is called the scalar moment of $\vec{F}$ about the axis $\hat{a}$.