Paper 1, TDC Part-1
Chapter– 2, Complex Algebra and J operator

By:

Mayank Mausam

Assistant Professor (Guest Faculty)

Department of Electronics

L.S. College, BRA Bihar University,

Muzaffarpur, Bihar

Complex Algebra and J operator

Polar Form of Vector Representation.

We can express $E(\cos\theta \pm j\sin\theta)$ in the ~~fi~~ simplified form of $E\angle\theta$, where E represent the magnitude of the vector and θ its inclination with X-axis.

For inclination in anticlockwise direction $E\angle\theta$ &

" " " clockwise " $E\angle-\theta$.

$E\angle\pm\theta$ is simply a short-hand or symbolic style of writting $Ee^{\pm j\theta}$.

Summarizing, we have the following alternate ways of representing ~~vector~~ complex quantities.

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(i) Rectangular Form → $\overrightarrow{OE} = a \pm jb$

(ii) Trignometrical form → $\overrightarrow{OE} = E(\cos\theta \pm j \cdot \sin\theta)$

(iii) Exponential form $\overrightarrow{OE} = E e^{\pm j\theta}$

(iv) Polar form (conventional) $\overrightarrow{OE} = E \angle \pm\theta$

Example (1) Write the equivalent exponential and polar forms of vector $8 + j6$. How will you illustrate the vector means of diagram

Solⁿ Given vector is $\overrightarrow{OP} = 8 + j6$

magnitude of $\overrightarrow{OP} = \sqrt{8^2 + 6^2} = \sqrt{64+36} = 10$

$\theta = \tan^{-1}\left(\frac{6}{8}\right) \approx 37°$

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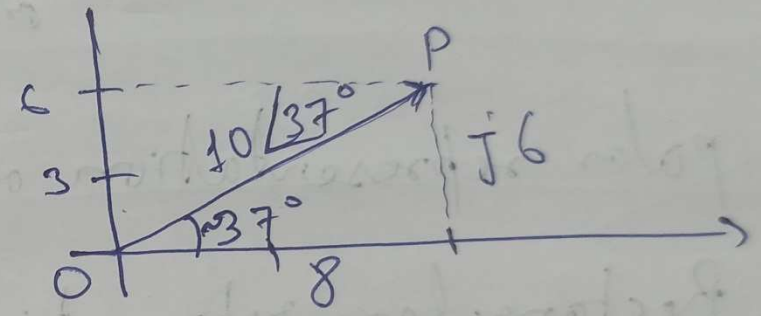
Equivalent exponential representation of ~~$\bar{OP} = 10 e^{j37°}$~~

$\bar{OP} = 10 e^{j37°}$

& Equivalent polar representation of

$\bar{OP} = 10 \angle 37°$

Vector diagram



Example 2 Repeat example 1 if $\bar{OP} = -5 + 12j$

Soln: Given vector $\bar{OP} = -5 + 12j$

magnitude of $\bar{OP} = \sqrt{(-5)^2 + (12)^2} = \sqrt{25+144} = 13$

$\tan^{-1}(-12/5) = (90° + 23°) \approx 113°$

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$\theta = \tan^{-1}\left(\frac{12}{-5}\right) = \tan^{-1}\left(-\frac{12}{5}\right) = (90° + 23°) \approx 113°$

Equivalent exponential representation of

$\bar{OP} = 13\, e^{j113°}$

& equivalent polar representation of

$\bar{OP} = 13 \angle 113°$

Example 3) A vector is represented by $20\, e^{-j(2\pi/3)}$. Write the various equivalent form of the vector and illustrate by means of a vector diagram, the magnitude and position of the given vector.

Sol) Angle of vector is $-2\pi/3 = -\frac{2\times 180°}{3} = -120°$

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So the vector is in 120° clockwise direction.

the vector is given in exponential form $20e^{-j2\pi/3}$

(i) its trignometric form is → $20[\cos(\frac{2\pi}{3}) - j\sin(2\pi/3)]$

or $20[\cos(-120°) - j\sin(-120°)]$

(ii) polar representation of vector is $20\angle -120°$

(iv) Rectangular representation :→

$x = 20\cos(-120°) = -10$

$y = 20\sin(-120°) = -17.32$

So the rectangular representation of vector is

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$(-10 - j17.32)$

Addition & Subtraction of Complex Quantities

Addition or subtraction of Complex Quantities are best carried in rectangular form.

Addition of vector :→

Consider 2 vector quantities

$\overline{OP} = x_1 + jy_1$ & $\overline{OQ} = x_2 + jy_2$

In addition the real component of different vectors will be added together and imaginary component will be added together. We will not add real component with imaginary or

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Vice-versa.

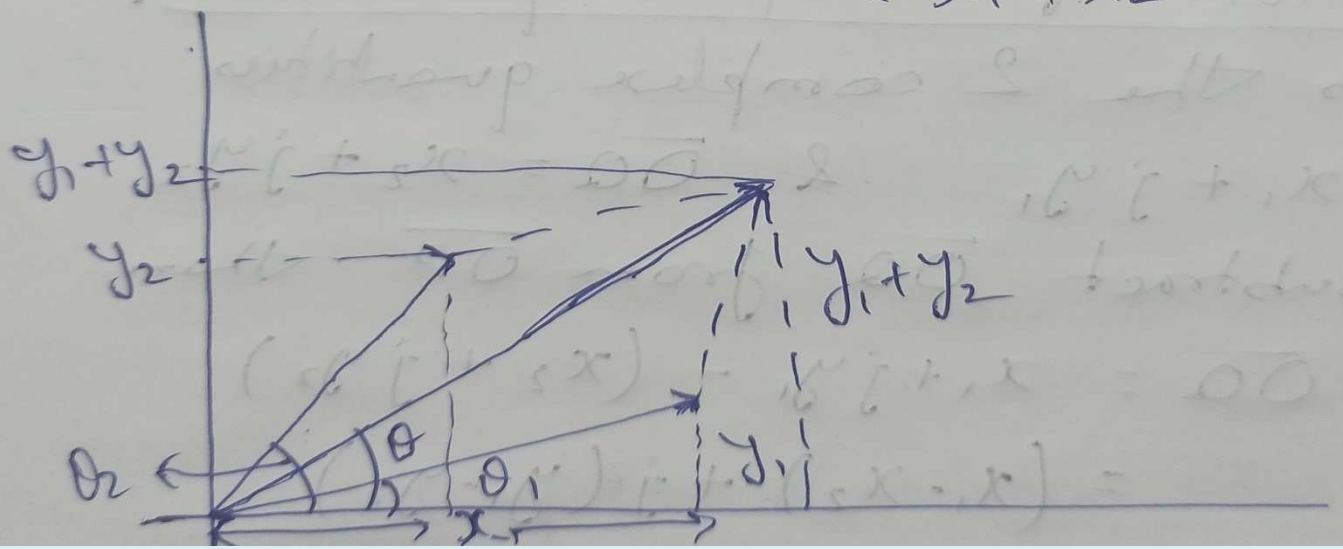
So $\overline{OP} + \overline{OQ} = x_1 + jy_1 + x_2 + jy_2$

$= (x_1 + x_2) + j(y_1 + y_2)$

Magnitude of resultant vector is $= \sqrt{(x_1+x_2)^2 + (y_1+y_2)^2}$

$= \sqrt{(x_1+x_2)^2 + (y_1+y_2)^2}$

Position of vector with respect to x-axis is given by $\theta = \tan^{-1}\left(\frac{y_1+y_2}{x_1+x_2}\right)$



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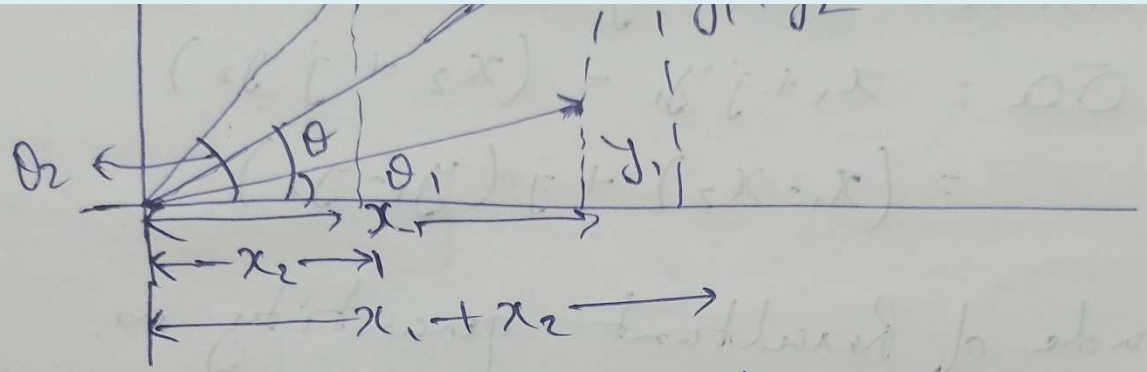

Graphical representation of the addition process.

Example: Add vector $\bar{e}_1 = 10 e^{j30^\circ}$ & $e_2 = 8 \angle -45^\circ$

Soln. Let us first convert the vector in rectangular form.

$$\bar{e}_1 = 10 e^{j30^\circ} = 10\cos(30^\circ) + j\,10\sin 30^\circ$$

$$= 8.66 + j5$$

$$\bar{e}_2 = 8\angle -45 = 8\cos(-45^\circ) + j\,8\sin(-45^\circ)$$

$$= 5.66 - j\,5.66 = 5.66(1 - j)$$

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So the resultant vector is given by

$\bar{E} = \bar{e}_1 + \bar{e}_2$

$= 8.66 + j5 + 5.66 - j5.66$

$= (8.66 + 5.66) + j(5 - 5.66)$

$\bar{E} = 14.32 - j0.66$

Subtraction of Complex quantities

Consider the 2 complex quantities

$\bar{OP} = x_1 + jy_1$ & $\bar{OQ} = x_2 + jy_2$

Now subtract $\bar{OQ}$ from $\bar{OP}$ then

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$$\overline{OP} - \overline{OQ} = x_1 + jy_1 - (x_2 + jy_2)$$

$$= (x_1 - x_2) + j(y_1 - y_2)$$

Magnitude of Resultant quantity is,

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Angle subtented by the given vector with respect to x-axis is given by,

$$\theta = \tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right)$$

Assign Subtract $\cdot E = 14.32 - j0.66$ from

$$F = -5.68 - j\,8.66$$

Also represent the resultant vector in different form.

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For any query contact- 9771474020

Thank You