Matrices

(ix) **Transpose of a matrix**: The matrix obtained interchanging rows and columns of a matrix, A is called the transpose of A and is denoted by A'

Thus: if $A = (a_{ij})_{m,n}$ then $A' = (a'_{ji})_{n,m}$
where $a'_{ji} = a_{ij}$

Ex :- $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$, $A' = [A^T] = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$ $(A')' A = A$

(x) **Symmetric matrix**: A matrix is called symmetric if $A' = A$. Note that a symmetric matrix must be square.

(xi) **Scalar product of a matrix**: If $A = (a_{ij})_{m,n}$ and K any number (i.e. scalar) then the scalar product of A with K is denoted by KA (or AK) and is defined by $KA = AK = (Ka_{ij})_{m,n}$

(xii) **Skew symmetric matrix**: A matrix A is called symmetric if $A' = -A$

Note that a skew symmetric matrix must be a square. Its diagonal elements should be zero.

(xiii) **Orthogonal matrix**: A square matrix A is called orthogonal if $A'A = AA' = I$ where I is the unit matrix.

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Symmetric matrices: A square matrix A is said to by symmetric if $A = A'$ i.e. the matrix is equal to its transpose.

example: $A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$, $A' = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

$\therefore A = A'$

Eg:- $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ → square matrix, $A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$

$\therefore A = A'$, $A \to a_{ij} = a_{ji}$

Here A is a square matrix and $A = A'$

$\therefore$ A is a symmetric matrix.

Skew symmetric matrix:- A square matrix A is said be skew symmetric if $A = -A'$, i.e. the matrix is equal to its transpose with (-ve) sign. Diagonal elements of skew symmetric matrix are all zero.

Eg:- $A = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & y \\ -1 & -y & 0 \end{bmatrix}$, $A' = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -y \\ 1 & y & 0 \end{bmatrix}$ $\therefore A = -A'$

$A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$, $A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$

$a_{ij} = -a_{ji}$
$a_{ii} = -a_{ii}$
$a_{12} = -a_{21}$
$2a_{ii} = 0$
$a_{ii} = 0$

Properties of transpose:-

(i) $(A')' = A$

(ii) $(A+B)' = A' + B'$

(iii) $K(A') = (KA)'$

* (iv) $\boxed{(AB)' = B'A'}$ ← Important property

(v)

Ex:- $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$A^T = A' = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

③

1. Theorem :- For any square matrix A with real number elements

① → $(A+A')$ is a symmetric matrix

② → $(A-A')$ is a skew symmetric matrix

eg :- $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ $A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

eg $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

$A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

$C = A + A' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$, $D = (A - A') = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$C' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} \therefore C = C'$ $D' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$\therefore D' = -D$

Proved

2. Theorem :- Any square matrix with real number entries) can be expressed as the sum of a symmetric and skew symmetric matrix.

$A = \underbrace{\frac{1}{2}(A+A')}_{\text{symmetric}} + \underbrace{\frac{1}{2}(A-A')}_{\text{skew symmetric matrix}}$ [from above theorem]

Eg:- $A = \begin{bmatrix} 4 & 7 \\ 8 & 2 \end{bmatrix}$ $A' = \begin{bmatrix} 4 & 8 \\ 7 & 2 \end{bmatrix}$

$A + A' = \underbrace{\begin{bmatrix} 8 & 15 \\ 15 & 4 \end{bmatrix}}_{\text{symmetric}}$ $A - A' = \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{skew symmetric matrix}}$ [from above theorem]

$\begin{bmatrix} 4 & 7 \\ 8 & 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 8 & 15 \\ 15 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 4 & 15/2 \\ \frac{15}{2} & 2 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1/2 & 0 \end{bmatrix}$

$= \begin{bmatrix} 4 & 7 \\ 8 & 2 \end{bmatrix}$

Proved