

Thermal Physics
Rctilinear flow of Heat
Lecture - 2

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Online Course Link :

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Rectilinear flow of Heat (Fourier Equation for flow of Heat) :

Let us consider a long metal bar of uniform cross –sectional area A , heated at one end .

The bar lies along the x – axis whose origin is at the hot end . Heat flows along this axis .

Now consider a thin slice with two parallel planes A and B perpendicular to the length of the bar at distances x and $x + \delta x$ from the hot end .

If θ be the temperature of the plane A

and $\frac{d\theta}{dx}$ be the heat which leaves the slice at B per second .

Then

$$Q = -K A \frac{d\theta}{dx} \quad (1)$$

$$Q - dQ = -K A \frac{d}{dx} \left(\theta + \frac{d}{dx} \delta x \right), \quad (2)$$

Where K is the thermal conductivity of the bar . The negative sign indicates that the temperature decreases with increases x .

Then the heat gained by the slice is

$$\begin{aligned}dQ &= -KA \frac{d\theta}{dx} - \left\{ -KA \frac{d}{dx} \left(\theta + \frac{d}{dx} \delta x \right) \right\}, \\ &= KA \frac{d^2\theta}{dx^2} \delta x .\end{aligned}\tag{3}$$

If dQ be the amount gained by the slice is used partly in raising the temperature of the slice and partly in radiation from the surface of the slice .

Let ρ be the density and s be the specific heat of the material of the bar , then mass of the slice is

$$= A\delta x \rho$$

Let E be the emissive power of the surface and θ be the average excess temperature of the surface over the surrounding, then according to Newton's Law of Cooling, the heat radiated per second from the surface is

$$dQ_2 = E\rho\delta x\theta \quad (5)$$

Hence we have

$$KA \frac{d^2 \theta}{dx^2} \delta x = A\delta x\rho s \frac{d\theta}{dt} + E\rho\delta x\theta$$

This can be written as,

$$\frac{d\theta}{dt} = h \frac{d^2 \theta}{dx^2} - \mu\theta, \quad (6)$$

Where

$$h = \frac{K}{\rho s} \quad ,$$
$$\mu = \frac{E}{A \rho s} \quad (7)$$

The constant h is known as the diffusivity of material , which determines the rate at which the temperature changes in a bar .

Equation (6) is the standard **Fourier equation for linear flow of heat** . If we neglect the heat lost by radiation , (6) reduces to

$$\frac{d\theta}{dt} = h \frac{d^2 \theta}{dx^2} \quad (8)$$

Steady state

The temperature remains constant in slice element , this state is named as steady state . Hence

$$\frac{d\theta}{dt} = 0$$

and (4) reduces to

$$d^2 \theta / dx^2 = \frac{\mu}{h} \theta = m^2 \theta , \quad (9)$$

$$\text{where } m^2 = \frac{\mu}{h} \theta = \frac{Ep}{KA}$$

Here the radiation is included .

It is a second order differential equation and can be solved to give

$$\theta = e^{\pm mx}$$

Hence complete solution of (9) is

$$\theta = A e^{mx} + B e^{-mx} \quad (10)$$

Where A and B are constants whose values are determined from the boundary conditions .

Let us assume

$$\theta = \theta_0 \quad \text{at} \quad x = 0$$

$$\theta = 0 \quad \text{at} \quad x = \infty$$

Then

$$\theta_0 = A + B$$

$$0 = A e^{\alpha}$$

But e^{α} is not zero and hence $A = 0$.

Thus the finding solution will be

$$\theta = \theta_0 e^{-mx} \quad (11)$$

When bar is covered, so that the radiation loss is neglected, (9) becomes

$$d^2 \theta / dx^2 = 0 \quad (12)$$

$$\theta = Ax + B \quad (13)$$

Where A and B are constants, which can be determined again from the boundary conditions.

If l be the length of the bar , then

$$\theta = \theta_0 \quad \text{at} \quad x = 0$$

$$\theta = \theta_1 \quad \text{at} \quad x = l$$

From (13) we get

$$B = \theta_0$$

$$A = \theta_0 - \theta_1 / l$$

And hence

$$\theta = \theta_0 - (\theta_0 - \theta_1 / l) x$$

Where θ is the temperature at any point x .