

# Theory of Equation (1)

## Polynomial equations:-

The general form of a polynomial equation of  $n^{\text{th}}$  degree is

$$a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots + a_{n-1} x + a_n = 0.$$

where  $a_0, a_1, a_2, \dots, a_n$  are constants.

The exponent of the highest power of  $x$  in a polynomial determines the degree of that polynomial equation.

For example,  $x^3 - 6x^2 + 11x - 6 = 0$  is a polynomial equation of the third degree.

$x^4 - 5x^3 + 6x^2 - 4x + 24 = 0$  is a polynomial equation of the fourth degree and so on.

Thus the general forms of cubic and biquadratic equations are respectively

$$a_0 x^3 + a_1 x^2 + a_2 x + a_3 = 0$$

$$\text{and } a_0 x^4 + a_1 x^3 + a_2 x^2 + a_3 x + a_4 = 0$$

Equations of third degree and fourth degree are called cubic and biquadratic respectively.

Similarly, equations of fifth and sixth degrees are called quintic and sextic respectively.

An equation is said to be complete when it contains terms of all powers in  $x$  and incomplete when some of the terms are absent i.e. when some one of the coefficients  $a_0, a_1, a_2, \dots$  etc. are zero.

Example:-  $x^3 + 3x^2 - 27x + 104 = 0$  is a complete cubic equation; whereas  $x^3 - 15x + 4 = 0$  is an incomplete equation because the term involving  $x^2$  is absent.

An equation is said to be numerical or algebraical according as its coefficients are numbers or algebraic quantities.

Example:- (i) The equation  $x^3 - 6x^2 + 11x - 6 = 0$  is a numerical equation;

(ii)  $a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$  is an algebraic equation.

Solution of an equation is meant the determination of all the roots which satisfy the given equation.

We can write an equation of  $n^{\text{th}}$  degree symbolically as  $f_n(x) = 0$  or by  $f(x) = 0$ .