Theory of Equations

Problems :-

Q.(1) Solve the equation $2x^3 - 15x^2 + 37x - 30 = 0$ whose roots are in A.P.

Sol:- Let $\alpha - \delta,\ \alpha,\ \alpha + \delta$ be the roots of the given equation

Then, $(\alpha - \delta) + \alpha + (\alpha + \delta) = -\frac{3b}{a}$

Then, we have $(\alpha - \delta) + \alpha + \alpha + \delta = \frac{15}{2}$

$\Rightarrow 3\alpha = \frac{15}{2} \quad \therefore \alpha = \frac{5}{2}$ ——— (1)

Also, $(\alpha - \delta)\alpha + (\alpha - \delta)(\alpha + \delta) + \alpha(\alpha + \delta) = \frac{37}{2}$

$\Rightarrow \alpha^2 - \alpha\delta + \alpha^2 - \delta^2 + \alpha^2 + \alpha\delta = \frac{37}{2}$

$\Rightarrow 3\alpha^2 - \delta^2 = \frac{37}{2} \Rightarrow 3 \cdot \frac{25}{4} - \delta^2 = \frac{37}{2}$

$\Rightarrow \delta^2 = \frac{75}{4} - \frac{37}{2} = \frac{1}{4}$

$\therefore \delta = \pm \frac{1}{2}$

$\therefore$ The roots are $\frac{5}{2} - \frac{1}{2},\ \frac{5}{2},\ \frac{5}{2} + \frac{1}{2}$

i.e. $2, \frac{5}{2}, 3$

Note :- If we take $\delta = -\frac{1}{2}$, we get the same roots.

of equations

Q. (2) Solve $x^3 - 15x^2 - 33x + 847 = 0$

Diminish the roots by 5 in order to reduce the equation to standard form and the transformed equation is

$$z^3 - 108z + 432 = 0$$

as shown below

$$\begin{array}{c|cccc}
5 & 1 & -15 & -33 & -847 \\
 & & 5 & -50 & -415 \\
\hline
 & & -10 & -83 & 432 \\
 & & 5 & -25 & \\
\hline
 & & -5 & -180 & \\
 & & 5 & & \\
\hline
 & & 0 & &
\end{array}$$

Let $z = u + v$; $\therefore z^3 - 3uvz - (u^3 + v^3) = 0$

Comparing $uv = 36$ and $u^3 + v^3 = -432$

$\therefore u^3$ and v^3 are the roots of $t^2 + 432t + 36^2 = 0$

or, $t^2 + 2 \cdot 216 \cdot t + (6^3)^2 = 0$

or, $(t + 6^3)^2 = 0$

$\therefore t = -6^3, -6^3$ i.e. $u = -6, v = -6$

$\therefore u + v = -12$ is a root of the z-cubic which when divided by $z + 12$ gives the quadratic.

$$z^2 - 12z + 36 = 0 \; ; \therefore (z - 6)^2 = 0$$

$\therefore$ roots of the z cubic are $6, 6, -12$ and hence of x-cubic are $11, 11, -7$ (two roots equal).