THEORY OF EQUATIONS (1)

SOLUTION OF CUBIC AND BIQUADRATIC EQUATIONS

Explain Cardon's method of solving the Cubic equation.

Let the Cubic equation be

$$a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0 \quad \text{——— (1)}$$

The above equation can be reduced to the form

$$z^3 + 3Hz + G = 0 \quad \text{——— (2)}$$

where $z = a_0x + a_1$

Now let us assume that $z = u+v$, Cubing both sides,

$$z^3 = u^3 + v^3 + 3uv(u+v)$$

or, $$z^3 - 3uvz - (u^3+v^3) = 0 \quad \text{——— (3)}$$

Comparing (2) and (3), we get

$$uv = -H \quad \text{——— (4)}$$

and $u^3 + v^3 = -G$

$\therefore u^3v^3 = -H^3$ and hence u^3 and v^3 are the roots of the quadratic

$$t^2 + Gt - H^3 = 0 \quad \text{——— (5)}$$

$$\therefore u^3 = \frac{-G + \sqrt{G^2 + 4H^3}}{2} \text{ and } v^3 = \frac{-G - \sqrt{(G^2 + 4H^3)}}{2}$$

Now from above we shall obtain three values of u and three of v; our root being $u+v$ will have therefore nine possible values because each of the three values of u is to be added to any of the three values of v. But there is a limitation imposed by the relation (4), i.e. $uv = -H =$ Constant

or, $v = \frac{-H}{u}$

Hence the values of u and v to be Combined should be such as to satisfy $uv = -H$. Now if we extract Cube root of u^3 we get $u, u\omega, u\omega^2$ and that of v^3 gives $v, v\omega, v\omega^2$.

Hence the three roots of the equation are

$$u+v,\ u\omega + v\omega^2,\ u\omega^2 + v\omega$$

where $\omega = \frac{-1+\sqrt{(-3)}}{2}$ and $\omega^2 = \frac{-1-\sqrt{(-3)}}{2}$

Having found z we can find the value of x by the relation $z = a_0 x + a_1$.