Theorem- A linear operator T on a finite dimensional inner product space V is unitary iff T preserves inner products.

Proof Let T be a linear operator on a finite dimensional inner product space V(F).

(i) suppose T is unitary

To prove that T preserves inner products

T is unitary $\Rightarrow$ T preserves inner products

(ii) Suppose T preserves inner products so that

$\langle T(x), T(y) \rangle = \langle x, y \rangle \quad \forall x, y \in V$

To prove that T is unitary, it is enough to show that.

② T is invertible

③ $\langle T(x), T(y) \rangle = \langle x, y \rangle \quad \forall x, y \in V$

Evidently (1) $\Rightarrow$ (3)

$\Rightarrow \langle T(x), T(x) \rangle = \langle x, x \rangle$

Now $T(x) = 0, x \in V$

$\Rightarrow \langle T(x), T(x) \rangle = \langle 0, 0 \rangle = 0$

$\Rightarrow \langle x, x \rangle = 0$

$\Rightarrow x = 0$

This proves that T is non-singular, so T is invertible