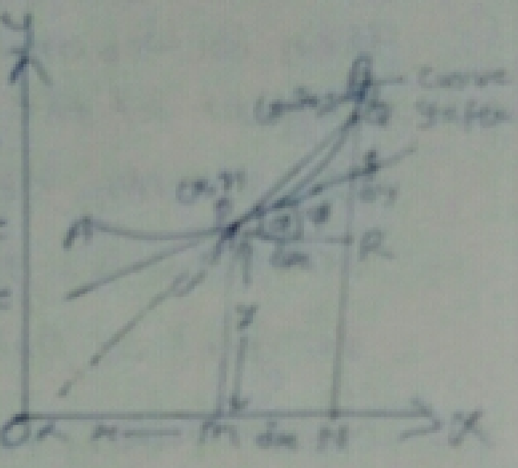
Tangent → Let p and Q be any two points on a given curve ($y=f(x)$).
When the point Q moves along the curve, concides with the point, when the limiting position of the secant pQ is called the tangent at the point p.



That is, a tangent is a straight line passing through two concident points p and Q.

Let the Equation of the curve be

$$y = f(x)$$

Let $p(x, y)$ [x_1, y_1] and $Q(x+\delta x, y+\delta y)$ [x_2, y_2] be two points on this curve.

The Equation of the line pQ is
Let (X, Y) denote the current co-ordinate

$$Y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(X - x_1)$$

$$Y - y = \frac{y + \delta y - y}{x + \delta x - x}(X - x)$$

$$Y - y = \frac{\delta y}{\delta x}(X - x) \quad \text{—— (1)}$$

From figure pM and QN are perpendiculars on the axis X. Then

$OM = x$, $ON = x + \delta x$ $\Rightarrow MN = \delta x$

$pM = y$, $QN = y + \delta y$ $\Rightarrow QR = \delta y$

Now we move the point Q along the curve to meet with the point P. i.e $Q \to P$, then $\delta x \to 0$ and also chord PQ becomes the tangent at the point P.

The limiting position of the equation (1) when $\delta x \to 0$ gives the equation of the tangent at point $P(x, y)$.

Now taking the limit of Eqn (1) as $\delta x \to 0$ we get

$$Y - y = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} (X - x)$$

$$Y - y = \frac{dy}{dx} (X - x)$$

This is the Eqn of the tangent at the point $P(x, y)$.

$\Rightarrow \frac{dy}{dx}$ is the slope of the tangent

$\therefore \frac{dy}{dx} = \tan\varphi$ $\qquad$ $\angle SPR = \varphi$
$\angle QPR = Q$

Thus if the tangent is parallel to x-axis then $\frac{dy}{dx} = 0$

* If we take (x, y) as current co-ordinates then the Eqn of the tangent at point (x_1, y_1) as $y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} (x - x_1)$

$\left(\frac{dy}{dx}\right)_{(x_1, y_1)}$ i.e put $x = x_1$ & $y = y_1$ in the diff. coeff.