* Symbolic Methods → Representing derivatives by powers of $D$.

Exp. Solve $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$

<u>Solution</u> Given that equation reduces to

$$(D^2+6D+9)y = 0$$

Then, the auxiliarly Equation is

$$D^2+6D+9=0$$

its equation compares with quadratic equation

$$ax^2+bx+c=0$$

$\because a=1 \quad b=6 \quad \& \ c=9$

$$D = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\therefore D = \frac{-6 \pm \sqrt{36-4\times1\times9}}{2\times1} = \frac{-6\pm0}{2} = -3\pm0$$

$D = -3$ and $-3$

or $D^2+6D+9=0$

$(D+3)^2 = 0 \Rightarrow (D+3)(D+3)=0$

roots are real and equal $D_1=D_2=-3$

$\therefore D = -3, -3$

Hence, the general solution is

$$y = (C_1 + C_2 x).e^{-3x}$$

Exp. Solve the linear differenential equation

$$\frac{d^4x}{dt^4} + 4x = 0$$

Solution:- Let $\frac{d}{dt} = D$ then given equation reduces

$$(D^4 + 4)x = 0$$

then auxiliarly Equation is

$$D^4 + 4 = 0$$

We can write this Equation as

$$D^4 + 4 + 4D^2 - 4D^2 = 0$$

$$(D^2+2)^2 - (2D)^2 = 0$$

$$(D^2 + 2 + 2D)(D^2 + 2 - 2D) = 0$$

Thus $D = \frac{-2 \pm \sqrt{4 - 4\times1\times2}}{2\times1}$ or $D = \frac{(+2) \pm \sqrt{4 - 4\times1\times2}}{2\times1}$

$D = \frac{-2 \pm \sqrt{-4}}{2}$ or $D = \frac{2 \pm \sqrt{-4}}{2}$

$D = \frac{-2 \pm 2i}{2}$ or $D = \frac{2 \pm 2i}{2}$

$D = -1 \pm i$ or $D = 1 \pm i$

$D_1 = -1 + i,\ D_2 = -1 - i,\ D_3 = 1 + i$ and $D_4 = 1 - i$

$\alpha_1 = -1$ & $\beta_1 = \pm 1$  $\alpha_2 = 1$ & $\beta_2 = \pm 1$

Hence the general solution of given Equation is

$$y = e^{-t}\cdot[c_1 \cos t + c_2 \sin t] + e^{t}\cdot[c_3 \cos t + c_4 \sin t]$$

## Inverse Operator: $\left\{\frac{1}{f(D)}\right\}$:-

(1) The $\frac{1}{f(D)}X$ is a function of $x$, not containting arbitrary constants then operated upon by $f(D)$ gives $X$.

i.e $f(D)\left\{\frac{1}{f(D)}X\right\} = X$

Thus $y = \frac{1}{f(D)}X$ satisfies the equation $f(D)y = X$ is its particular solution.

(2) $\frac{1}{D}X = \int f(x)\,dx$

Let $\frac{1}{D}X = y$

Operating by $D$ i.e $\frac{d}{dx} \to D$

$D.\frac{1}{D}X = Dy$ i.e $X = \frac{dy}{dx}$

Integrating both sides we get $y = \int X\,dx$

$$\boxed{\frac{1}{D}X = \int X\,dx}$$

(3) $\frac{1}{D-a}X = e^{ax}\int X e^{-ax}\,dx$

Exp. find $\frac{1}{D^2+2D-15} e^{2x}$

Sol. Given that

$$\frac{1}{D^2+2D-15} e^{2x}$$

$$\frac{1}{(D+5)(D-3)} \cdot e^{2x}$$

$a=3$

from $\frac{1}{D-a} \cancel{x} = e^{ax} \int x e^{-ax} dx$

then $\frac{1}{(D+5)} \cdot e^{3x} \int e^{-3x} \cdot e^{2x} dx$

$\Rightarrow \frac{1}{(D+5)} \cdot e^{3x} \int e^{-x} dx$

$\Rightarrow \frac{1}{(D+5)} \cdot e^{3x} \cdot (e^{-x} \cdot (\frac{1}{-1}))$

$\Rightarrow -\frac{1}{(D+5)} \cdot e^{2x}$

$a=-5$

$\Rightarrow -e^{-5x} \cdot \int e^{5x} \cdot e^{2x} dx$

$\Rightarrow -e^{-5x} \cdot \int e^{7x} dx$

$-e^{-5x} \cdot e^{7x} \cdot \frac{1}{7}$

$\Rightarrow -\frac{1}{7} \cdot e^{2x}$