Abstract Algebra

SUBGROUPS OF A GROUP

Let $(G, *)$ be a group and H is any subset of G such that $H \neq \phi$. Then by properties of a group

$$\forall \; a, b \in H \Rightarrow a, b \in G \text{ for } H \subset G$$
$$\Rightarrow a * b \in G \Rightarrow a * b \in H \text{ for } a*b \notin H$$

If $a * b \in H$, then we say that H is stable for the composition in G and the composition in G has induced a composition in H. Now, there are two possibilities.

(i) H is itself a group with respect to the operation

(ii) H is not a group with respect to

Complex of a Group :-

Any non-empty subset H of G is called a complex of the group G.

Subgroups :- A non-empty subset H of a group G is called a subgroup of G if H itself a group with respect to the operation defined in G.

Remarks :- (i) The two subgroups G and $\{e\}$ of the group G are called improper (or trivial) subgroups of G. Any subgroups other than these two subgroups is called a proper (or non-trivial) subgroups.

(ii) It is clear that, if H is a subgroup of G and K is a subgroup of H, then K is subgroup of G.

(iii) Every subgroup of G is a complex of G, but every complex is not always a subgroup.

Some Examples of subgroup:

(i) $[\{1, -1\}, \cdot]$ is a subgroup of $[\{1, -1, -i, -i\}, \cdot]$

(ii) $(Z, +)$ is a subgroup of $(Q, +)$

(iii) $(Q, +)$ is a subgroup of $(R, +)$

(iv) The set of all non-singular matrices with real elements whose determinants are 1, it is a subgroup of multiplicative group of all non-singular $n \times n$ matrices.

(v) The multiplicative group of positive rational numbers is a subgroup of the multiplicative group of all non-zero rational numbers.