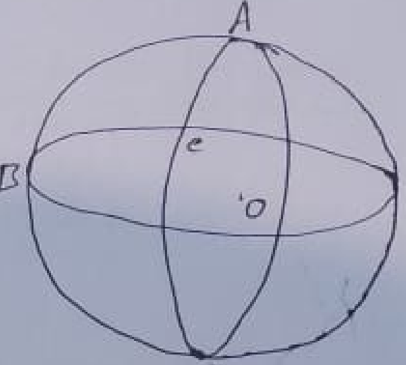
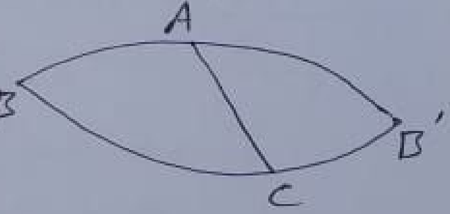
Q.) The sum of the three sides of
2018 a spherical triangle is less than the circumference of a great circle i.e. 360°.

Sol:- We produce the sides BA and BC to meets at B'.



$\therefore BAB' = BCB' = 180°$ as two great circles bisect each other.

Now from $\Delta AB'C$ we know that by ⓐ



$$AB' + CB' > AC$$

We add BA + BC to both sides.

$$(BA + AB') + (BC + CB') > BA + BC + AC.$$

or, $BAB' + BCB' > AB + BC + CA$

or, $180 + 180 > a + b + c$

or, $a + b + c < 360°$

2.) For any spherical triangle ABC, prove that $\sin\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \cdot \sin c}}$

Proof :- We have

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$$

Now we know that

$$2\sin^2 A/2 = 1 - \cos A$$

or, $2\sin^2\frac{A}{2} = 1 - \frac{\cos a - \cos b \cos c}{\sin b \sin c}$

or, $2\sin^2\frac{A}{2} = 1 - \frac{\cos a - \cos b \cos c}{\sin b \sin c}$

or, $2\sin^2\frac{A}{2} = \frac{(\cos b \cos c + \sin b \sin c) - \cos a}{\sin b \sin c}$

or, $2\sin\frac{A}{2} = \frac{\cos(b-c) - \cos a}{\sin b \sin c}$

or, $2\sin^2\frac{A}{2} = \frac{2\sin\frac{b-c+a}{2}\sin\frac{a-b+c}{2}}{\sin b \sin c}$

If we take that $2s = a+b+c$

then $a+b-c = a+b+c-2c = 2(s-c)$

and $a+c-b = a+b+c-2b = 2(s-b)$;

$\therefore \sin^2\frac{A}{2} = \frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}$

$\sin\frac{A}{2} = \sqrt{\left[\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}\right]}$ Proved