Exp. Solve the Homogeneous differential Equation

$$2x^3 y' = y(2x^2 - y^2)$$

Solution Given that H.D.E.

$$2x^3 y' = y(2x^2 - y^2)$$

$$y' = \frac{y}{2x^3}(2x^2 - y^2)$$

$$\frac{dy}{dx} = \frac{y}{x} - \frac{1}{2}\frac{y^3}{x^3} \quad \text{——} \quad (1)$$

Putting $y = vx \Rightarrow \frac{dy}{dx} = v + x.\frac{dv}{dx}$

$$\Rightarrow \frac{y}{x} = v$$

Then

$$v + x\frac{dv}{dx} = v - \frac{1}{2}v^3$$

$$x\frac{dv}{dx} = -\frac{1}{2}v^3$$

$$\frac{dv}{v^3} = -\frac{1}{2}\frac{dx}{x} \Rightarrow -2\frac{dv}{v^3} = \frac{dx}{x}$$

Integrating

$$-2\int\frac{dv}{v^3} = +\frac{1}{\cancel{2}}\int\frac{dx}{x} + \log c$$

$$-\frac{2v^{-3+1}}{-3+1} = +\frac{1}{\cancel{2}}\log x + \log c$$

$$-2\frac{v^{-2}}{-2} = +\cancel{\tfrac{1}{2}}\log x + \log c$$

$$v^{-2} = \log cx \Rightarrow \frac{x^2}{y^2} = \log cx$$

$$x^2 = y^2.\log cx$$

$\Rightarrow x^2 = y^2 \log cx$

$x = \pm y\sqrt{\log cx}$

Solve these Equations

Q1. $xy' = y\cos(\log y/x)$

Q.2 $y' = 2xy/(3x^2 - y^2)$

Q.3 $y^2 + x^2y' = xyy'$

Exp. Solve the homogeneous diff. Equation.

$xy' - y = x\tan y/x$ and $xy' = y - e^{y/x}$

Solution Given that H.D.E

$xy' - y = x\tan\frac{y}{x}$

$y' = \frac{y}{x} + \tan y/x$ —— ①

Putting $y = vx$ in Eq ①

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = v + \tan v$

$x\frac{dv}{dx} = \tan v$

$\frac{dv}{\tan v} = \frac{dx}{x}$

Integrating $\int\frac{dv}{\tan v} = \int\frac{dx}{x} + \log c$

$\int \cot v\,dv = \log x + \log c$

$\log\sin v = \log cx$

$\sin v = cx \Rightarrow \sin\frac{y}{x} = cx$