GROUP THEORY

Self Conjugate :- An element a of a group G is called self Conjugate element iff $C_a = \{a\}$

i.e. iff C_a Consists of the single element a

i.e. iff $x^{-1}ax = a \quad \forall \; x \in G$

$\Rightarrow$ iff $ax = xa \quad \forall \; x \in G$

Notes :-

(i) A self Conjugate element is one which Commutes with every element of the group.

(ii) The transform of a self Conjugate element remains the same.

(iii) Self Conjugate element is also called invariant element.

Conjugate Subgroup :- Transformation of an element by another element gives us a Conjugate element. Similarly, transformation of a subgroup by another subgroup gives Conjugate subgroups. Thus if A and B are two subgroups of a group G, then B is said to be Conjugate to A if there exists an element $x \in G$ such that $B = x^{-1}Ax$. The symbol "$B \leq A$" is read as B is Conjugate to A.

Theorem 1:- The normalizer $N(a)$ of $a \in G$ is a subgroup of G.

Proof :- By definition, we have

$$N(a) = \{x \in G : ax = xa\}$$

Let $x_1, x_2 \in N(a)$, then $ax_1 = x_1 a$, $ax_2 = x_2 a$

To show

$$x_1 x_2^{-1} \in N(a)$$

Firstly we shall show that $x_2^{-1} \in N(a)$

We Consider $ax_2 = x_2 a$

$$\Rightarrow \quad x_2^{-1}(ax_2)x_2^{-1} = x_2^{-1}(x_2 a)x_2^{-1}$$

$$\Rightarrow x_2^{-1} a = ax_2^{-1} \Rightarrow x_2^{-1} \in N(a)$$

Now to show

$$x_1 x_2^{-1} \in N(a)$$

We Consider $a(x_1 x_2^{-1}) = (ax_1)x_2^{-1} = (x_1 a)x_2^{-1} = x_1(ax_2^{-1})$

$$= x_1(x_2^{-1}a) = (x_1 x_2^{-1})a.$$

$$\therefore x_1 x_2^{-1} \in N(a)$$

$$\Rightarrow x_1 x_2 \in N(a) \Rightarrow x_1 x_2^{-1} \in N(a)$$

Hence, $N(a)$ is a subgroup of G.

we have