* Second-order differential Equation with Constant coefficient

Exp. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2$

Solution Given differential equation in symbo[lic] form

$$(D^2 - 4D + 4)y = x^2$$

To find C.F from $(D^2 - 4D + 4)y = 0$

Then, the auxiliarly Equation

$$D^2 - 4D + 4 = 0$$

$$(D-2)^2 = 0 \Rightarrow D = 2, 2$$

Thus, the C.F $= (C_1 + C_2 x)e^{2x}$.

To find P.I for x^2

$$P.I = \frac{1}{(D^2 - 4D + 4)} \cdot x^2$$

$$= \frac{1}{(D-2)^2} \cdot x^2$$

$$= \frac{1}{4\left(1 - \frac{D}{2}\right)^2} \cdot x^2$$

$$= \frac{1}{4}\left(1 - \frac{D}{2}\right)^{-2} \cdot x^2$$

$$= \frac{1}{4}\left(1 + 2\frac{D}{2} + 3\frac{D^2}{4} + \cdots\right)x^2$$

From $(1-D)^{-2} = 1 + 2D + 3D^2 \ldots$

$$P.I = \frac{1}{4}\left(x^2 + Dx^2 + \frac{3}{4}D^2x^2 + \cdots\right)$$

$$= \frac{1}{4}\left(x^2 + \frac{d}{dx}x^2 + \frac{3}{4}\frac{d^2}{dx^2}(x^2) + \cdots\right) \quad \because D = \frac{d}{dx}$$

$$= \frac{1}{4}\left(x^2 + 2x + \frac{3}{4}\cdot 2 + 0\right)$$

$$P.I = \frac{1}{4}\left(x^2 + 2x + \frac{3}{2}\right)$$

Hence, the general solution is

$$y = C.F + P.I$$

$$y = (c_1 + c_2x)e^{2x} + \frac{1}{4}\left(x^2 + 2x + \frac{3}{2}\right)$$

<u>Exp</u> Solve $\frac{d^2y}{dx^2} + y = x^3$

<u>Solution</u> given diff. Equ. reduces in symbolic form $(D^2+1)y = x^3$

To find C.F from $(D^2+1)y = 0$

then the auxiliary Equation $(D^2+1) = 0$

$$\Rightarrow D^2+1 = 0$$

$$\Rightarrow D = \pm i$$

$$C.F = c_1\cos x + c_2\sin x$$

Again find $P.I = \frac{1}{D^2+1}\cdot x^3$

$$P.I = (1+D^2)^{-1} x^3$$

$$= (1 - D^2 + D^4 - D^6 + \cdots) x^3$$

$$= x^3 - \frac{d^2}{dx^2}(x^3) + \frac{d^4}{dx^4}(x^3) - \cdots$$

$$= x^3 - 3\frac{d}{dx} x^2 + 3\frac{d^3}{dx^3} x^2 - 0$$

$$= x^3 - 6x + 6\frac{d^2}{dx^2} \cdot x - 0$$

$$= x^3 - 6x + 6 \cdot 1 \frac{d}{dx}(1) - 0$$

$$P.I = x^3 - 6x$$

Hence, the solution is

$$y = C.F + P.I$$

$$y = c_1 \cos x + c_2 \sin x + \underline{\underline{x^3 - 6x}}$$