Rules for finding the particular Solution of Integral

Let the Equation

$$\frac{d^n y}{dx^n} + k_1 \frac{d^{n-1} y}{dx^{n-1}} + k_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots k_n y = X$$

the symbolic form is

$$(D^n + k_1 D^{n-1} + \dots - k_n) y = f(D) y = X$$

Therefor, particular Integral is

$$P.I = \frac{1}{D^n + k_1 D^{n-1} + \dots k_n} X = \frac{1}{f(D)} X .$$

(a) Case I → when $X = e^{ax}$

→ if $f(a) \neq 0$, then $\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} . e^{ax}$

→ if $f(a) = 0$ then $\frac{1}{f(D)} e^{ax} = x \frac{1}{f'(a)} . e^{ax}$, $f'(a) \neq 0$

→ if $f(a) = 0$ and $f'(a) = 0$, then $\frac{1}{f(D)} e^{ax} = x^2 . \frac{1}{f''(a)} e^{ax}$,

& $f''(a) \neq 0$

(b) Case-II → when $X = \sin(ax+b)$ or $\cos(ax+b)$

→ if $f(-a^2) \neq 0$ then $\frac{1}{f(D^2)} \sin(ax+b) = \frac{1}{f(-a^2)} \sin(ax+b)$

→ if $f(-a^2) = 0$ then $\frac{1}{f(D^2)} \sin(ax+b) = x . \frac{1}{f'(-a^2)} . \sin(ax+b)$ and $f'(a^2)$

→ if $f(-a^2) = 0$ and $f'(-a^2) = 0$ then $\frac{1}{f(D^2)} \sin(ax+b) = x^2 \frac{1}{f''(-a^2)} \times$

$\sin(ax+b)$, and $f''(a^2) \neq 0$

Similarly for $X = \cos(ax+b)$.

Exp. Solve the Equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$$

Solution:- Given differenential equation is non-homogeneous. So general solution is $y = C.F + P.I$

The complementory factor of the homogen equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

The symbolic form of the equction

$$(D^2 - 5D + 6)y = 0$$

Then auxiliarly equation is

$$D^2 - 5D + 6 = 0 \Rightarrow f(D) = D^2 - 5D + 6$$

$$(D-2)(D-3) = 0$$

$$\Rightarrow (D-2) = 0 \text{ or } (D-3) = 0$$

$$D = 2, 3$$

Thus $C.F = c_1 e^{2x} + c_2 e^{3x}$

and $P.I = \frac{1}{f(D)} \cdot X = \frac{1}{D^2 - 5D + 6} \cdot e^{4x}$

$$= \frac{1}{16 - 20 + 6} \cdot e^{4x} \quad \left[\because \frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} \right.$$

$$P.I = \frac{1}{2} \cdot e^{4x}$$

$y = C.F + P.I \Rightarrow y = c_1 e^{2x} + c_2 e^{3x} + \frac{1}{2} e^{4x}$

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Exp. Solve $6\frac{d^2y}{dx^2}+25\frac{dy}{dx}+14y=\sin 3x$

Sol: Given differential equation is non-homogeneous. So general solution is $y = C.F + P.I$

To find complementary function (C.F) from Eqn.

$$6\frac{d^2y}{dx^2}+25\frac{dy}{dx}+14y=0$$

Let $D=\frac{d}{dx}$, then this equation reduces to

$$(6D^2+25D+14)y=0$$

The auxiliary equation is $6D^2+25D+14=0$

$\Rightarrow (3D+2)(2D+7)=0$

$\Rightarrow D=-2/3$ if $3D+2=0$

$D=-7/2$ if $2D+7=0$

Thus. $C.F = c_1 e^{-\frac{2}{3}x} + c_2 e^{-7/2 x}$.

To find particular integral $f(x) = \sin 3x$

$$P.I = \frac{1}{f(D)}.X = \frac{1}{6D^2+25D+14}.\sin 3x$$

$$= \frac{1}{6(-9)+25D+14}.\sin 3x$$

$\because \frac{1}{f(D^2)}\sin ax = \frac{1}{f(a^2)}\sin ax$

$$= \frac{1}{-54+25D+14}\sin 3x$$

$$P.I = \frac{1}{5(5D-8)}.\sin 3x$$

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$$P.I = \frac{1 \times (5D+8)}{5(5D-8)(5D+8)} \cdot \sin 3x$$

$$= \frac{1}{5} \cdot \frac{5D+8}{(25D^2 - 8^2)} \cdot \sin 3x$$

$$= \frac{1}{5} \frac{(5D+8) \cdot \sin 3x}{[25(-9) - 64]}$$

$$= \frac{1}{5(-225-64)} (5D \sin 3x + 8 \sin 3x)$$

$$= \frac{1}{5(-289)} \quad 5 \cos 3x \cdot (3) + 8 \sin 3x$$

$$P.I = -\frac{1}{1445} (15 \cos 3x + 8 \sin 3x)$$

Hence, general solution is

$$y = c_1 e^{-\frac{2}{3}x} + c_2 e^{-\frac{7}{2}x} - \frac{1}{1445} (15 \cos 3x + 8 \sin 3x)$$