RINGS

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Definition :- Let R be a non-empty set. An algebraic structure $(R, '+', '\cdot')$ together with two binary operations addition and multiplication for all $a, b \in R$ is called a ring if this structure satisfies following properties:

(i) closed under addition:

$$a+b \in R \quad \forall\, a, b \in R$$

(ii) Associative under addition:

$$a+(b+c) = (a+b)+c, \ \forall\, a, b, c \in R$$

(iii) Existence of identity:

$$0+a = a = a+0, \ \forall\, a \in R$$

then 0 is an additive identity of R.

(iv) Existence of inverse:

there exists an element $-a \in R$ such that

$$-a+a = 0 = a+(-a), \ \forall\, a \in R$$

then $-a$ is additive inverse of a.

(v) Commutative under addition:

$$a+b = b+a, \ \forall\, a, b \in R$$

(vi) closed under multiplication:-

$$a \cdot b \in R \quad \forall\, a, b \in R$$

(vii) **Associative under multiplication :-**

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c \quad \forall a, b, c \in R$$

$$a \cdot (b + c) = a \cdot b + a \cdot c \text{ (left distributive)}$$

and $(b + c) \cdot a = b \cdot a + c \cdot a$ (Right distributive)

OR

An algebraic structure $(R, +, \cdot)$ is said to be a ring provided $a+b \in R$, $a \cdot b \in R$ for all $a, b \in R$ and satisfies following properties :

(i) $(R, '+')$ is an abelian group

(ii) Multiplication is associative.

(iii) Multiplication is distributive over addition.

Ring with Unity :

Definition :- A ring R is said to be ring with unity if the multiplicative identity i.e. $1 \in R$ such that

$$1 \cdot a = a = a \cdot 1 \quad \forall a \in R$$

Commutative Ring :

Definition :- A ring R is said to be a Commutative ring if

$$a \cdot b = b \cdot a, \quad \forall a, b \in R.$$