* Relaxtion Method :-

Consider the epuations

$$a_1x+b_1y+c_1z=d_1 \quad \text{--- (i)}$$
$$a_2x+b_2y+c_2z=d_2 \quad \text{--- (ii)}$$
$$a_3x+b_3y+c_3z=d_3 \quad \text{--- (iii)}$$
$\Big\}$ — (1)

We define the residuals R_x, R_y, R_z by relations

$$R_x = d_1-(a_1x+b_1y+c_1z)$$
$$R_y = d_2-(a_2x+b_2y+c_2z)$$
$$R_z = d_3-(a_3x+b_3y+c_3z)$$
$\Big\}$ — (2)

To start with we assume $x=y=z=0$ and calculate the initial residuals. Then the residuals are reduced step by step by giving increments to be the variables. For this purpose, we construct the following operation table-

	δR_x	δR_y	δR_z
$\delta x=1$	$-a_1$	$-a_2$	$-a_3$
$\delta y=1$	$-b_1$	$-b_2$	$-b_3$
$\delta z=1$	c_1	$-c_2$	$-c_3$

We note from the epuations (2) that if x is increased by 1 (keeping y and z constant). R_x, R_y, R_z decrease by a_1, a_2, a_3 respectively. This is shown in the above table alongwith the effects on the residuals when y and z are given unit increments. (The table is the transpose of the coefficient matrix).

At each step, the numerically largest residual is reduced to almost zero. To reduce a particular

- residual, the value of the corresponding variable is changed, e.g. to reduce R_x by p, x should be increased by p/a_1.

When all the residuals have been reduced to almost zero, the increments in x, y, z are added separately to give the desired solution.

Exp. Solve, by Relaxation method the Equation

$$\left.\begin{aligned} 9x - 2y + z &= 50 \quad \text{---(i)} \\ x + 5y - 3z &= 18 \quad \text{---(ii)} \\ -2x + 2y + 7z &= 19 \quad \text{---(iii)} \end{aligned}\right\} \text{---(1)}$$

Solution The residuals are given by

$$\left.\begin{aligned} |-9| > |2| + |-1| \rightarrow R_x &= 50 - 9x + 2y - z \quad \text{---(i)} \\ |-5| > |1| + |3| \rightarrow R_y &= 18 - x - 5y + 3z \quad \text{---(ii)} \\ |-7| > |2| + |-2| \quad R_z &= 19 + 2x - 2y - 7z \quad \text{---(iii)} \end{aligned}\right\} \text{---(1)}$$

The operation table is

Increment of table	δR_x	δR_y	δR_z
$\delta x = 1$	-9	-1	2
$\delta y = 1$	2	-5	-2
$\delta z = 1$	-1	3	-7
$x = y = z = 0$	50	18	19

$\delta x = \frac{50}{9} = 5.55 = 5$, $y = 0$, $z = 0$ putting in Eqn (i) of (1)

The relaxation table is

	R_x from (i)	R_y from (ii)	R_z from (iii)	
from (i-iii) $x=y=z=0$	50 (i)	18 (ii)	19 (iii)	(i)
$(50/9=) \Rightarrow \delta x=5$	5 (i)	13	29 (iii)	(ii)
$(29/7) \Rightarrow \delta z=4$	1	25	1	(iii)
$\delta y=5$	11	0	-9	(iv)
$\delta x=1$	2	-1	-7	(v)
$\delta z=-1$	3	-4	0	(vi)
$\delta y=-0.8$	1.4	0	1.6	(vii)
$\delta z=0.23$	1.17	0.69	-0.9	(viii)
$\delta x=0.13$	0	0.56	0.17	(ix)
$\delta y=0.112$	0.224	0	-0.054	(x)

$$\Sigma \delta x = 5+1+0.13 = 6.13$$

$$\Sigma \delta y = 5+(-0.8)+0.112 = 4.31$$

$$\Sigma \delta z = 4-1+0.23 = 3.23$$

Thus. $x = 6.13$

$y = 4.31$

$z = 3.23$