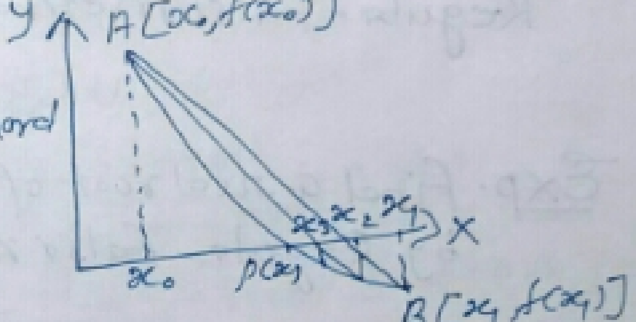
* Regula-falsi Method :-

This is the oldest method of finding the real roots of the polyno-mial equation $f(x)=0$ and closely resembles the bisection method.

Here we choose two points x_0 and x_1 such that $f(x_0)$ and $f(x_1)$ are of opposite signs i.e the graph of $y=f(x)$ crosses the x-axis between these points (in figure). This indicates that a root lies between x_0 & x_1 consequently $f(x_0)>$ $f(x_1)<0$



Equation of the chord joining the points $A[x_0, f(x_0)]$ and $B[x_1, f(x_1)]$ is

$$y-f(x_0)=\frac{f(x_1)-f(x_0)}{x_1-x_0}(x-x_0) \longleftarrow (1)$$

Let the point of intersection of the chord (AB) with the x-axis (i.e $y=0$) as an approximation to the root. So the abscissa of the point where the chord cuts the x-axis ($y=0$) is given by

$$x_2=x_0-\frac{x_1-x_0}{f(x_1)-f(x_0)}\cdot f(x_0) \longrightarrow (2)$$

which is an approximation to the root.

If now $f(x_0)$ and $f(x_2)$ are of opposite signs, then the root lies between x_1 and x_2. So replacing x_1 by x_2 in (2) we obtain the next approximation x_3.
(The root could as well lie between x_1 & x_2 and we would obtain x_3 accordingly).
This procedure is required to repeated till the root is found to desired accuracy.
The iteration process based on Eqn (1) is known as the method of false position or Regula-Falsi method.

Exp. find a real root of the equation $x^3-2x-5=0$ by Regula-Falsi method.

Solution Given that Equation

$$f(x) = x^3 - 2x - 5$$

Let $f(2) = 8 - 2\times 2 - 5 = -1$

$f(3) = 27 - 2\times 3 - 5 = 16$

i.e root lies between 2 and 3

Taking $x_0 = 2,\ x_1 = 3 \Rightarrow f(x_0) = -1$

$f(x_1) = 16$

putting in the method of false position we get -

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0) \text{ ——— } (1)$$

$$= 2 - \frac{3-2}{16-(-1)}(-1)$$

$$x_2 = 2 + \frac{1}{17} = 2.0588$$

Now

$$f(x_2) = f(2.0588) = -0.3908$$

i.e the next root lies between 2.0588 and 3

Then taking $x_0 = 2.0588$ & $x_1 = 3$

$$\Rightarrow f(x_0) = -0.3908$$

$$f(x_1) = 16$$

and putting in Eqn (1) we get

$$x_3 = 2.0588 - \frac{(3-2.0588)}{16-(-0.3908)}(-0.3908)$$

$$x_3 = 2.0588 + \frac{0.9412}{16.3908}(.3908) = 2.0813$$

Now

$$f(x_3) = f(2.0813) = (2.0813)^3 - 2\times 2.0813 - 5$$

$$= 9.0157955278 - 4.1626 - 5$$

$$f(x_3) = -0.1468044922$$

$\because$ taking $x_0 = 2.0813$ and $x_1 = 3$ then

$$f(x_0) = -0.1468$$

$$f(x_1) = 16$$

and putting in Eqn (1), we get

next approximation

$$x_4 = 2.0813 - \frac{(3-2.0813)}{16-(-0.1468)}(-0.1468)$$

$$x_4 = 2.0898 \Rightarrow f(x_4) = -0.5587$$

Repeating this process, the succesive approximations are

$x_5 = 2.0915 \Rightarrow f(x_5) = -0.034$

$x_6 = 2.0934 \Rightarrow f(x_6) = -0.01284$

$x_7 = 2.0941 \Rightarrow f(x_7) = -0.00504$

$x_8 = 2.0943 \Rightarrow f(x_8) = -0.00281$

$x_9 = 2.0945 \Rightarrow f(x_9) = -0.00169$ & $x_{10} = 2.0945$

$\Rightarrow f(x_{10}) = -0.00057$

Hence the root is 2.0945

Que- Find the root of the equation $2x - \log_{10} x = 7$, which lies between 3.5 and 4 by Regula-falsi method.