DEFINITIONS :-

(i) **SET** :- A set is defined as a collection of well defined elements i.e. It is a collection of objects which are distinct and distinguishable.

(ii) **SEQUENCE** :- Let S be a set of numbers. A succession of numbers $a_1, a_2, a_3, \ldots, a_n \ldots$ all $\in S$ formed according to some definite rule, then the set is called a sequence which is usually denoted by $\{U_n\}$.

(iii) **SERIES** :- The sum of elements of a sequence is called series.

e.g. (i) $1+2+3+ \ldots$

(ii) $u_1 + u_2 + \ldots$ etc

(iv) **Finite series** :- If a series has a finite number of terms, the series is called a finite series.

(v) **Infinite series** :- A series having an infinite no. of terms is called an infinite series.

Here $u_1 + u_2 + u_3 + \cdots + u_3 + \cdots$ is an infinite series. It is denoted by $\sum_{n=1}^{\infty} u_n$ or by simply $\sum u_n$.

The sum to n terms of this series is denoted by S_n

$$\therefore S_n = u_1 + u_2 + u_3 + \cdots + u_n$$

But the sum of an infinite series is the series

$$\lim_{n \to \infty} S_n = u_1 + u_2 + u_3 + \cdots + u_n + \cdots$$

(v) Divergent series :- If $\lim_{n\to\infty} S_n = \infty$ $\sum U_n$ is said to diverge. If $\lim_{n\to\infty} S_n = -\infty$ $\sum U_n$ is said to diverge to $-\infty$.

(vi) Oscillatory series :- If $\lim_{n\to\infty} \inf. S_n \neq \lim_{n\to\infty} \sup S_n$ we say that the series $\sum u_n$ is oscillatory. In short we say that $\sum U_n$ oscillate if $\lim_{n\to\infty} S_n$ does not exist.

(vii) **Convergent series** :- The series $\sum U_n$ is said to converge if $\lim_{n\to\infty} S_n$ (where $S_n = U_1 + U_2 + \dots + U_n$) is finite.

This means that if for a fixed finite number l, there exists corresponding to any given number $\epsilon > 0$ a positive integer m such that

$$|S_n - l| < \epsilon \quad \forall\ n \geq m.$$

then $\sum U_n$ is said to converge to "sum" l.

(ix) **limit** :- The sequence $\{U_n\}$ is said to have the limit l if for a given positive number ϵ there exists a positive integer m such that $|U_n - l| < \epsilon$ for all integral values of $n \geq m$.

If $\{U_n\}$ has the limit l. we shao it by writtin $U_n \to l$ as $n \to \infty$.

or, $\lim_{n\to\infty} U_n = l$.

Cauchy's general principle of Convergence

The necessary and sufficient Condition for the Convergence of the series $\sum U_n$ is that given any $\epsilon > 0$ there exists a positive integer m such that

$$|U_{n+1} + U_{n+2} + \dots + U_{n+p}| < \epsilon \quad \forall\ n \geq m \text{ and } \forall \text{ positive integer } p.$$

Proof:- Let

$$S_n = u_1 + u_2 + u_3 + \dots + u_n.$$

Then $\sum U_n$ Converges $\Leftrightarrow$ S_n (Converges by defn)

$\Leftrightarrow$ $\forall\ \epsilon > 0\ \exists$ a integer m such that $|S_{n+p} - S_n| < \epsilon$ $\forall\ n \geq m$, $\forall$ positive integer p (by Cauchy's general principle) of Convergence of a sequence)

$\Leftrightarrow$ $\forall\ \epsilon > 0$, $\exists$ an integer m s.t. $|U_{n+1} + U_{n+2} + \dots + U_{n+p}| < \epsilon$ $\forall\ n \geq m$ $\forall$