* Quasihomogeneous Equations:-

A function $f(x,y)$ is called quasihomogene[ous] of degree k if under the definite α and β

$$f(t^{\alpha}x, t^{\beta}y) = t^{k}f(x,y)$$

A differential equation $\frac{dy}{dx} = f(x,y)$ —

is called quasihomogeneous of degree $\beta - \alpha = k$, that is

$$f(t^{\alpha}x, t^{\beta}y) = t^{\beta-\alpha}f(x,y) \quad \text{—(2)}$$

for Exp. $\quad f(x,y) = \dfrac{x^6 - 2y^4}{x^4 y}$

we have $\alpha = 2$ and $\beta = 3$, since

$$\frac{(t^2x)^6 - (t^3y)^4}{(t^2x)^4(t^3y)} = t^{3-2}\,\frac{x^6 - 2y^4}{x^4y}$$

$$\Rightarrow \frac{t^{12}x^6 - t^{12}2y^4}{t^8x^4t^3y} = \frac{t^{12}(x^6 - y^4)}{t^{11}x^4y}$$

$$= t\,\frac{x^6 - y^4}{x^4y}$$

The form of a quasihomogeneous equation

$$f(t^{\alpha}x, t^{\beta}y) = t^{\beta-\alpha} f(x,y)$$

suggests that it may be simplified by introducing a new variable, which will denote by u.

$$y = u x^{\beta/\alpha}$$

and equation (1) $\frac{dy}{dx} = f(x,y)$ becomes separable.

<u>Exp.</u> Show that the equation is quasihomogeneous, and solve it. <u>p-17</u>

$$f(x) = 2x\,dy + (x^2y^4+1)\,y\,dx = 0$$

<u>Solution</u> Given equation is writing as

$$\frac{dy}{dx} = -\frac{y(x^2y^4+1)}{2x}, \quad x \neq 0$$

We have $\quad f(x,y) = -\dfrac{y(x^2y^4+1)}{2x} \quad$ — ①

Looking on $f(x,y)$ as quasihomogeneous function we must find α and β putting $x = t^{\alpha}x$ & $y = t^{\beta}y$ in Eq (1) we get

$$\frac{t^{\beta}y(t^{2\alpha}x^2t^{4\beta}y^4+1)}{2t^{\alpha}x} = t^{\beta-\alpha}\frac{y(x^2y^4+1)}{2x}$$

or $$\frac{y(x^2y^4t^{2\alpha+4\beta-\alpha+\beta}+t^{\beta-\alpha})}{2x} = t^{\beta-\alpha}\frac{y(x^2y^4+1)}{2x}$$

Whence $2\alpha+4\beta-\alpha+\beta = \beta-\alpha$

$$2\alpha+4\beta=0$$

$$\alpha+2\beta=0$$

The equation being quasihomogeneous ($\beta/\alpha=-\frac{1}{2}$) we set

$$y=ux^{-1/2}$$

$$\frac{dy}{dx} = -\frac{1}{2}x^{-3/2}u + x^{-1/2}\frac{du}{dx}$$

The resulting equation is

$$x^{-1/2}\frac{du}{dx} - \frac{1}{2}x\,x^{-3/2}u = -\frac{x^{-1/2}u(u^4+1)}{2x}$$

$$\frac{du}{dx} = \frac{u^5}{2x}$$

$$\frac{dx}{x} = -2\frac{du}{u^5}, \quad u\neq 0$$

Integrating both sides

$$\log|x| + \log|c| = \frac{u^{-4}}{2}$$

$$\log|xc| = \log_e e^{(u^{-4}/2)}$$

$$cx = \exp(u^{-4}/2) \quad \text{Putting } u = yx^{1/2}$$

$$cx = \exp(x^{-2}y^{-4}/2)$$