Now following cases may arise

Case (I) If $M_J\, g\, \mu_B\, H / KT << 1$ i.e , $T >> 1$

and

$H << 1$ or for high temperature and for low field strength ,

$$M = N \sum_{-J}^{+J} M_J\, g\, \mu_B\, (1 + M_J\, g\, \mu_B\, H / KT) / \sum_{-J}^{+J} (1 + M_J\, g\, \mu_B\, H / KT)$$

$$M = N \left[g \mu_B \sum_{-J}^{+J} M_J + g^2 \mu_B^2 H / KT \sum_{-J}^{+J} M_J^2 \right] / \sum_{-J}^{+J} (+1) + g \mu_B H / KT \sum_{-J}^{+J} M_J \qquad (4)$$

We know that

$$\sum_{-J}^{+J} M_J = 0$$

$$\sum_{-J}^{+J} M_J^2 = J(J+1)(2J+1) / 3$$

$$\sum_{-J}^{+J} M_J = 2J + 1$$

Putting these values in (4) , we get

$$M = N[\, 0 + g^2 \mu_B^2 H J(J+1)(2J+1) / 3KT \,] / (2J+1) + 0$$

$$M = Ng^2 J(J+1)\mu_B^2/3KT \qquad (5)$$

Now the paramagnetic susceptibility χ is given by

$$\chi = M/H = Ng^2 J(J+1)\mu_B^2/3KT \qquad (6)$$

From classical theory the paramagnetic susceptibility is given by

$$\chi = N \mu^2 / 3KT$$

The eq.(6) is very similar to the classical result . The eq.(6) can be written as

$$\chi = N\mu_J^2/3KT \qquad (7)$$

Where $\mu_J^2 = g^2 J (J+1) \mu_B^2$

$$\mu_J = g\mu_B \sqrt{(J(J+1)} \qquad (8)$$

we may also write

$\mu_J = P_{eff}\mu_B$ (9)

Where $P_{eff} = g\sqrt{(J(J+1)}$ (10)

= Effective number of Bohr magneton

Case (2) . If $M_J\, g\, \mu_B\, H/\, KT \gg 1$

i.e , $T \ll 1$ and $H \gg 1$

or for low temperature and for high field strength , the term

$M_J\, g\, \mu_B\, H/ KT$ is not less than one and it is impossible to make a series

expansion of (3).

If we put $x = g\, J\, \mu_B\, H/ KT$, the form of (3) after algebraic manipulation

becomes

$$M = N\, g\, J\, \mu_B\, B_J(x) \qquad (11)$$

Where $B_J(x)$ is the Brillouin function and is given by

$$B_J(x) = (2J +1/ 2J) \coth [(2J +1)x/2J] – 1/2J \cot h\, (x/2J) \qquad (12)$$