Matrix

Properties of adjoint and inverse

(c) show that if the inverse of a matrix exists, then it is unique.

Let A be a non-singular matrix. Then its inverse exists. Let B be an inverse of A. If possible, let C be another inverse of A. Then

$$AB = BA = I \quad (1)$$

$$\text{and } AC = CA = I \quad (2)$$

$$\therefore \quad C = CI = C(AB) \text{ by } (1)$$

$$= (CA)B \text{ (associativity of product of matrices)}$$

$$= IB \text{ by } (2)$$

$$= B$$

Hence, if there are two inverses of A, then they are equal.

$\therefore$ Inversely, if it exists, is unique.

(d) **Reversal rule** :- If A and B are non-singular (square) matrices of the same order then

$$(AB)^{-1} = B^{-1}A^{-1}$$

Proof :- Let A and B be non-singular (square) matrices of the same order.

Then

$|A| \neq 0, \quad |B| \neq 0$

$\therefore |AB| = |A||B| \neq 0$

$\therefore AB$ is a non-singular (square) matrix of order n.

$\therefore (AB)^{-1}$ exists and is a non-singular (square) matrices of order n.

Also, since A, B are non-singular (square) matrices of order n.

$\therefore A^{-1}, B^{-1}$ exists and non-singular (square) matrices of order n.

$\therefore (AB)(B^{-1}A^{-1})$ and $(B^{-1}A^{-1})(AB)$ are both defined.

Now $(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1}$ (associativity of product of matrices)

$= A(I_n)A^{-1}$ where I_n is unit matrix of order n.

$= (AI_n)A^{-1}$

$= AA^{-1}$

$= I_n$ (1)

similarly, $(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = (B^{-1}I_n)B$

$= B^{-1}B = I_n$ (2)

(1) and (2) mean that $B^{-1}A^{-1}$ is the inverse of AB

i.e. $B^{-1}A^{-1} = (AB)^{-1}$