(c) If A is a non-singular matrix, show that $(A')^{-1} = (A^{-1})'$

<u>Proof</u> :- since A is non-singular

$$\therefore \ |A| \neq 0$$

$$\therefore \ |A'| = |A| \neq 0$$

$\therefore$ A' is non singular and hence $(A')^{-1}$ exists

Also since A is non-singular,

$\therefore$ A^{-1} exists.

Also since A is non-singular,

$\therefore$ A^{-1} exists.

Now we have the identities

$AA^{-1} = I = A^{-1}A$ (where I is the unit matrix of the same order as A)

Taking transposes, we get

$$(AA^{-1})' = I' = (A^{-1}A)'$$

or $$(A^{-1})' \cdot A' = I = A'(A^{-1})'$$

This shows that $(A^{-1})'$ is the inverse of A'.

i.e. $$(A^{-1})' = (A')^{-1}$$

(f) If the product of two non-zero square matrices of the same order be a zero (ie. null) matrix, show that both of them must be singular matrices.

Let A and B be two non-zero square matrices of order n. Let O be the null matrix of order n.

It is given that $AB = O$ —— (1)

If possible, suppose that B is non-singular, then B^{-1} exists. Post multiplying (1) by B^{-1} we get

$$(AB)B^{-1} = OB^{-1} = O$$

or, $A(BB^{-1}) = O$

or, $AI_n = O$, where I_n is the unit matrix of order n.

or, $A = O$

This contradicts the data that A is non-zero.

∴ B cannot be non-singular

∴ B is singular.

similarly A is singular.

(g) Prove that $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$ where A, B, C are non-singular matrices of the same order.

First we prove that if A and B are non-singular matrices of the same order n. Hence $(AB)^{-1}$ exists; and $(AB)^{-1} = B^{-1}A^{-1}$

Then we add

we have

$$
\begin{aligned}
(ABC)^{-1} &= ((AB)C)^{-1} \\
&= C^{-1}(AB)^{-1} \\
&= C^{-1}(B^{-1}A^{-1}) \\
&= C^{-1}B^{-1}A^{-1}
\end{aligned}
$$

Proved