//

Matrices

(vi) Properties of transposition

(a) $(A')' = A$ where A is any matrix;

(b) $(A+B)' = A' + B'$ (provided $A+B$ is defined);

(c) $(AB)' = B'A'$ (reversal law) provided AB is defined.

<u>Proof</u> :- (a) Let $A = (a_{ij})_{m,n}$ be any matrix then by the definition of transpose of a matrix.

$$\text{Then } A' = (a'_{ji})_{n,m} \text{ where } a'_{ji} = a_{ij}$$

$$\therefore (A')' = (a''_{ij})_{m,n} \text{ where } a''_{ij} = a'_{ji} = a_{ij}$$

$$\therefore (A')' = (a_{ij})_{m,n} = A$$

(b) Let $A = (a_{ij})_{m,n}$ and $B = (b_{ij})_{m,n}$ so that $A+B$ is defined.

and $A+B = (a_{ij} + b_{ij})_{m,n} = (c_{ij})_{m,n}$

where $c_{ij} = a_{ij} + b_{ij}$

$$\therefore (A+B)' = (c'_{ji})_{n,m} \text{ where}$$

$$c'_{ji} = c_{ij} = a_{ij} + b_{ij} \quad \text{——— (1)}$$

Now $A' = (a'_{ji})_{n,m}$ where $a'_{ji} = a_{ij}$

and $B' = (b'_{ji})_{n,m}$ where $b'_{ji} = b_{ij}$ $\Big\}$ 2

clearly $A' + B'$ is defined and

$$A' + B' = (a'_{ji} + b'_{ji})_{n,m}$$

$$= (a_{ij} + b_{ij})_{n,m} \text{ by } 2$$

$$= (c'_{ji})_{n,m} \text{ by } 1$$

$$= (A+B)' \text{ by } 1$$

c Let $A = (a_{ij})_{m,n}$ and

$B = (b_{jk})_{n,p}$

so that AB is defined and

$$AB = \left(\sum_{j=1}^{n} a_{ij}\, b_{jk}\right)_{m,p} = (c_{ik})_{m,p}$$

where $c_{ik} = \sum_{j=1}^{n} a_{ij}\, b_{jk}$

$$\therefore (AB)' = (c'_{ki})_{p,m} \text{ where } c'_{ki} = c_{ik}$$

$$= \sum_{j=1}^{n} a_{ij} b_{jk}$$

——— 1

Now $B' = (b'_{ij})_{p,n}$ where $b'_{kj} = b_{jk}$

and $A' = (a'_{ji})_{n,m}$ where $a'_{ji} = a_{ij}$ ②

② shows that $B'A'$ is defined and

$$B'A' = \left(\sum_{j=1}^{n} b'_{kj} \cdot a'_{ji}\right)_{p,m}$$

$$= \left(\sum_{j=1}^{n} b_{jk} a_{ij}\right)_{p,m}$$

$$= \left(\sum a_{ij} b_{jk}\right)_{p,m}$$

($\because$ Product of scalars is commutative)

$$= (c'_{ki})_{p,m} \quad [\text{by } ①]$$

$$= (AB)' \quad [\text{by } ①]$$