

Matrices

①

(vi) Properties of transposition

(a) $(A')' = A$ where A is any matrix;

(b) $(A+B)' = A' + B'$ (provided $A+B$ is defined);

(c) $(AB)' = B'A'$ (reversal law) provided AB is defined.

Proof :- (a) Let $A = (a_{ij})_{m,n}$ be any matrix then by the definition of transpose of a matrix.

Then $A' = (a'_{ji})_{n,m}$, where $a'_{ji} = a_{ij}$

$\therefore (A')' = (a''_{ij})_{m,n}$, where $a''_{ij} = a'_{ji} = a_{ij}$

$\therefore (A')' = (a_{ij})_{m,n} = A$

(b) Let $A = (a_{ij})_{m,n}$ and $B = (b_{ij})_{m,n}$ so

that $A+B$ is defined.

and $A+B = (a_{ij} + b_{ij})_{m,n} = (c_{ij})_{m,n}$

where $c_{ij} = a_{ij} + b_{ij}$

$\therefore (A+B)' = (c'_{ji})_{n,m}$ where

$c'_{ji} = c_{ij} = a_{ij} + b_{ij}$ ——— ①

Now $A' = (a'_{ji})_{n,m}$ where $a'_{ji} = a_{ij}$ }
 and $B' = (b'_{ji})_{n,m}$ where $b'_{ji} = b_{ij}$ } ⁽²⁾

clearly $A' + B'$ is defined and

$$A' + B' = (a'_{ji} + b'_{ji})_{n,m}$$

$$= (a_{ij} + b_{ij})_{n,m} \text{ by } (2)$$

$$= (c'_{ji})_{n,m} \text{ by } (1)$$

$$= (A+B)' \text{ by } (1)$$

(c) Let $A = (a_{ij})_{m,n}$ and

$$B = (b_{jk})_{n,p}$$

so that AB is defined and

$$AB = \left(\sum_{j=1}^n a_{ij} b_{jk} \right)_{m,p} = (c_{ik})_{m,p}$$

$$\text{where } c_{ik} = \sum_{j=1}^n a_{ij} b_{jk}$$

$$\therefore (AB)' = (c'_{ki})_{p,m} \text{ where } c'_{ki} = c_{ik}$$

$$= \sum_{j=1}^n a_{ij} b_{jk}$$

(1)

Now $B' = (b'_{ij})_{p,n}$ where $b'_{kj} = b_{jk}$ } (3)
and $A' = (a'_{ji})_{n,m}$ where $a'_{ji} = a_{ij}$ } (2)

(2) shows that $B'A'$ is defined and

$$B'A' = \left(\sum_{j=1}^n b'_{kj} \cdot a'_{ji} \right)_{p,m}$$

$$= \left(\sum_{j=1}^n b_{jk} a_{ij} \right)_{p,m}$$

$$= \left(\sum_{j=1}^n a_{ij} b_{jk} \right)_{p,m}$$

(\because Product of scalars is commutative)

$$= (c'_{ki})_{p,m} \quad [\text{by } \textcircled{1}]$$

$$= (AB)' \quad [\text{by } \textcircled{1}]$$