Properties of matrix operations :-

(2) (i) show that matrix addition is Commutative

Let $A = (a_{ij})_{m,n}$ and $B = (b_{ij})_{m,n}$ be matrices of the same type. Then $A+B$ and $B+A$ are both defined.

We have to show that $A+B = B+A$

Now, $A+B \stackrel{\text{def.}}{=} (a_{ij}+b_{ij})_{m,n}$ [$\because$ addition

$= (b_{ij}+a_{ij})_{m,n}$ of scalars is Commutative]

$\stackrel{\text{def.}}{=} B+A$

Hence matrix addition is Commutative.

(ii) show that matrix addition is associative

Let $A = (a_{ij})_{m,n}$, $B = (b_{ij})_{m,n}$ and $C = (c_{ij})_{m,n}$ be matrices of same type. Then

$A+B$, $(A+B)+C$, $B+C$ and $A+(B+C)$ are all defined. we have to show that

$$(A+B)+C = A+(B+C)$$

Now, $A+B \stackrel{\text{def.}}{=} (a_{ij}+b_{ij})_{m,n}$

and $(A+B)+C \stackrel{\text{def.}}{=} [(a_{ij}+b_{ij})+c_{ij}]_{m,n}$

$= [a_{ij}+(b_{ij}+c_{ij})]_{m,n}$

[$\because$ addition of scalars is associative]

$$= (a_{ij})_{m,n} + (b_{ij} + c_{ij})_{m,n}$$

$$\stackrel{\text{df.}}{=} A + (B + C)$$

Hence the result.

(iii) Show that matrix multiplication is not Commutative.

Proof :- In order that the matrix multiplication be Commutative. both the following Conditions should be satisfied for any two matrices A and B :-

(a) If the product AB is defined then the product BA is also defined; and

(b) $AB = BA$

We note that if A is an $m \times n$ matrix and B is an $n \times p$ matrix $(p \neq m)$; then AB is defined but BA is not defined.

∴ In condition (a) is not satisfied for all matrices A and B.

Moreover, as seen in the following example, even if AB and BA are both defined, the condition (b) may not be satisfied :-

Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

Thus we see that whenever $B+C$ and $A(B+C)$ are defined.

Then AB, AC and $AB+AC$ are also defined.

This proves (a)

Finally, we have

$$AB + AC = \left(\sum_{j=1}^{n} \{a_{ij} b_{jk} + a_{ij} c_{jk}\}\right)_{m,p}$$

[by (2) and (3)]

$$= \left\{\sum_{j=1}^{n} a_{ij} (b_{jk} + c_{jk})\right\}_{m,p}$$

[∵ scalar multiplication is distributive over scalar addition

$$= A(B+C) \text{, by (1)}$$

This proves (b).