Problems on Equipotential surfaces

Q. ① show that the family of surfaces defined by x^2+y^2 = Constant can be a family of equipotential surfaces in free space and find the law of potential.

The given family is $x^2+y^2=\lambda$ (constant) —— ①

Differentiating ① partially w.r.to x we get

$$\frac{\partial\lambda}{\partial x} = 2x$$

similarly $\frac{\partial\lambda}{\partial y} = 2y$

$$\therefore \frac{\partial^2\lambda}{\partial x^2} = 2, \quad \frac{\partial^2\lambda}{\partial y^2} = 2$$

$$\therefore \frac{\frac{\partial^2\lambda}{\partial x^2} + \frac{\partial^2\lambda}{\partial y^2}}{\left(\frac{\partial\lambda}{\partial x}\right)^2 + \left(\frac{\partial\lambda}{\partial y}\right)^2} = \frac{4}{4(x^2+y^2)} = \frac{1}{x^2+y^2} = \frac{1}{\lambda}$$

= a function of λ alone.

$\therefore$ ① is a possible family of equipotential surfaces.

Second part :– The potential V of the family is given by

$$V = f(\lambda) \quad \text{—— ②}$$

where $\frac{f''(\lambda)}{f'(\lambda)} = -\frac{1}{\lambda}$

Integrating we get $\log f'(\lambda) = \log A - \log \lambda$

$$\text{or, } f'(\lambda) = \frac{A}{\lambda}$$

Integrating again,

$$V = f(\lambda) = B + A \log \lambda$$

where A, B are constants.